PHYSICS

A Textbook for Higher Secondary Schools

Classes XI-XII

PART II

(Volume 1)

Editorial Board

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Foreword

This book is for one-semester course in Physics for Class XII for the academic stream in the 10+2 pattern of education A companion book for another semester is expected to be out shortly

The main features of the book are functionality, conceptual clarity and a disciplinary approach suitable for those who will pursue higher education in the subject I hope it will be found useful

I am grateful to the members of the Editorial Board, the authors and the editors for preparing this manuscript and working on it till its printing through the press was over. The work has been done in a short period of time requiring considerable hard labour for which I am grateful

Any suggestion for improvement of the book is welcome

SHIB K MITRA
Director
National Council of
Educational Research and Training

New Delhi
March 1978

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UNIT 11

Thermodynamics

We are already familiar with many topics in heat such as properties of gases, change of states, calorimetry, etc. We have yet to study many other topics such as heat in relation to mechanical work, radiation by hot bodies, pyrometry, etc. All such topics fall under the scope of thermodynamics. The detailed study of these topics is beyond the scope of this Unit. However, we will now try to study some general principles of thermodynamics and their applications to some familiar situations. So we begin with the development of topics which are pre-requisites for this study.

11.1 Temperature

Consider two bodies, A and B. Body A be such that it feels cold to the hand, and body B be such that it feels hot to the hand. We then say that the body B is at a higher temperature than the body A. Let the two be kept in contact with each other. After some time we will find that both A and B give the same temperature sensation, and we then say that the two are in thermal equilibrium with each other.

The test for the thermal equilibrium of two bodies is to use a third body which should be such that it does not appreciably disturb the thermal state of either of the bodies.* Suppose body A is in thermal equilibrium with a body C, say a thermometer, and body B is also in thermal equilibrium with body C, the thermometer, then A and B are in thermal equilibrium with one another. This statement is known as the Zeroth Law of Thermodynamics.

11.2 Heat Energy

Drop a hot body in a vessel containing cold water. The water warms up till the temperature of the two becomes the same. This change in temperature is due to the fact that energy flows from the hot body to cold body. This flow of energy from one body to the other, which takes place by virtue of their temperature difference alone, is called heat energy, or just heat for short. The hot body and the cold water, if considered separately, do not have any heat energy though they do have energy. Heat energy becomes evident only when energy flows from one to the other. When the two attain a common temperature; the quantity of heat energy transferred will be zero and the two systems will be in thermal equilibrium. In

*This means the heat capacity, i.e., mass into specific heat, of the test body should be much smaller than that of the bodies whose temperature it has to measure.

thermodynamics, an adiabatic* process is one in which there is no transfer of heat energy.

Conventionally, the quantity of heat energy transferred to a body is said to be positive $(+Q)$, and that transferred from a body is said to be negative $(-Q)$.

Till recently, the unit of heat energy was the calorie. It is usually defined as the quantity of energy required to raise the temperature of 1 g of water through 1°C. However, it has recently been internationally agreed to express all energy changes in the energy unit, the joule (J). A joule is defined as the work done when a force of one newton acts through one metre. The relation between the two is : 1 calorie =4.18 joules. If the amount of heat energy supplied to a system is 5 calories, then Q= 20.9J

11.3 Work

Consider a cylinder having a movable piston (Fig. 11.1). Let it contain one gram mole of a perfect gas. If the pressure exerted by the gas be P and the area of cross-section of the cylinder be A, then the force exerted by the gas on the piston will be PA. If due to this force the piston moves an infinitesimal distance dl, then the corresponding work done will be

$$dW = PA\, dl$$
$$= PdV$$

where dV is the infinitesimal volume change Adl. What will be the work done if the piston moves a finite distance? Consider the case when the piston steadily moves through a finite distance and the volume of the gas increases from V_i to V_f. If the process is carried out slowly, W will be given by

$$W = \int_{V_i}^{V_f} PdV \qquad \dots (11.1)$$

Is this work done a positive or a negative quantity? We make use of the following convention.

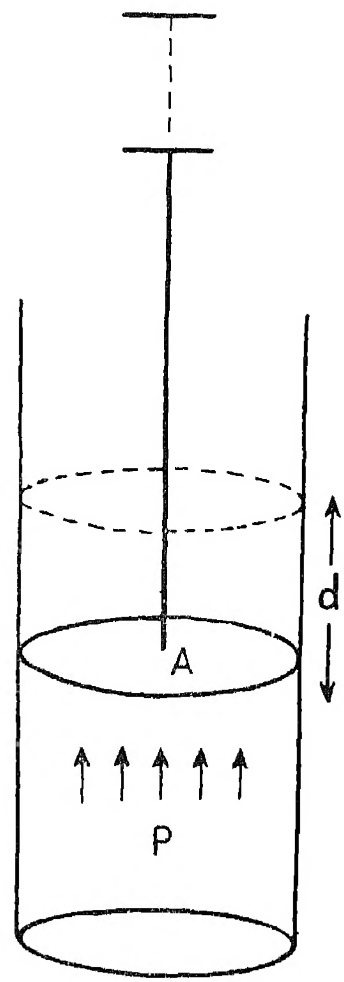


Fig. 11.1 Expansion of gas inside a cylinder

*a=not, dia=through, bates=heat
Adiabatic is a Greek word meaning heat not passing through.

In thermodynamics, an object or a group of objects under consideration is referred to as a system and everything external to the system is called the surroundings.

So in our discussion the cylinder with the piston containing gas constitutes the system. Conventionally, the work done by a system is taken to be positive.

Let us consider the converse process Suppose the gas is compressed by adding weights on the piston; then the work done W' will be

$$W' = \int_{V'_i}^{V'_f} P'\, dV$$

Where p' is the pressure exerted on the gas. In this case the work is done on the system and conventionally W' will be negative.

The above concepts are illustrated in the following example.

EXAMPLE 11.1

Three moles of an ideal gas kept at a constant temperature of 300k are compressed from a volume of 4 litres to a volume of 1 litre Calculate the work done in the process.

$$W = \int_{V_i}^{V_f} P dV$$

Since PV=nRT, for an ideal gas, we get

$$W = \int_{V_i}^{V_f} \frac{nRT\, dV}{V}$$

$$= nRT \log_e \frac{V_f}{V_i}$$

Since n=3 mole, T=300 degree,

R=8.31 J/mole deg.

and $\log_e x = 2.3 \log_{10} x$ we get

$$W = 3 \times 8\ 31 \times 300 \times 2\ 3 \log_{10} 1/4$$

$$= -10320\ J$$

The negative sign indicates that work is done on the system.

In the above we considered the process of expansion of the gas and calculated the work done in the process We used the relation PV=nRT. The analytical method would fail where the relationship between P and V is not known In such cases the same can be done graphically as described below.

Indicator diagram

Consider the system in which the gas exerts pressure on the piston of the cylinder (Fig. 11.1) Let the volume of the gas increase very slowly In order to represent the process graphically, let the volume V be plotted along the abscissa and the pressure P along the ordinate. By plotting the values of P and V for successive positions of the piston, the process can be represented by a curve as in Fig. 11.2.

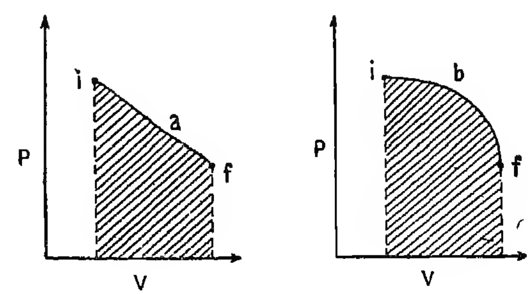


Fig. 11.2 Indicator diagram

The area under the curve gives the work done in the process, PdV. Such a diagram in which pressure is plotted along the ordintae

and volume along the abscissa is called a P-V diagram, or an indicator diagram.

11.4 Work and Heat

We have seen that both heat and work are measured in the same units, namely joule. So the question arises what is the relation between the two quantities. Joule was one of the early scientists who experimentally studied this. He proceeded as follows

A rod to which some paddles are attached was mounted in a cylinder containing water. The rod could be revolved by a thread and pulley arrangement as shown in Fig. 11.3. The

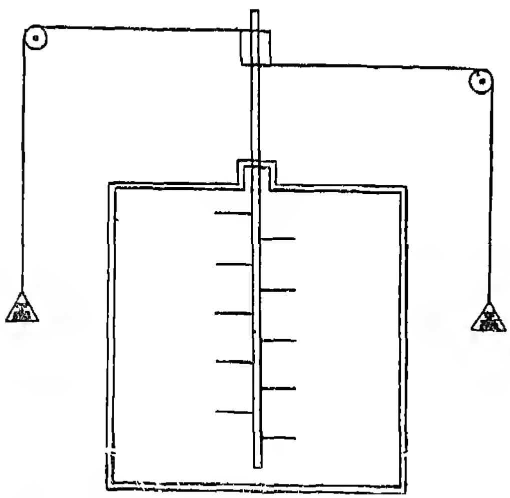


Fig. 11.3 Apparatus for Joule's experiment

water got heated on revolving the rod. The quantity of heat gained by the water was calculated by noting the rise in temperature.

The quantity of work done in revolving the rod was also calculated, by noting the distance through which the weights fell. By applying necessary corrections for the heat energy lost in radiation, frictional losses, etc., Joule concluded that a definite relation exists between the work done and the heat gained. Mathematically, he expressed this relation as

$$W = JH \qquad ..(11.2)$$

where W is the work done in joules, H is the heat gained in calories and J is a constant. J is called the mechanical equivalent of heat and from his experiment Joule determined its value as 4.18 joules per calorie.

So we see from above that heat and work are inter-related and all of the work done can be converted into heat. What about the converse process? Can all of heat supplied be converted to work? We will discuss this later.

11.5 Internal Energy

We know that work and heat are both some form of energy and the two are inter-convertible. Let us consider the case of compression of a gas enclosed in a cylinder with a moving piston When the piston is pushed by doing some work, the gas gets compressed and its temperature rises. If the walls of the system are thermally insulated (Fig. 11.4) no heat will be transferred i.e the process will be adiabatic. The work done in the system gets converted into another form of energy which is called the internal energy of the system and the total energy remains conserved Most of the internal energy is due to the kinetic energy arising from the motion of the molecules inside the system. When a thermodynamic system changes from one state to another, its internal energy also changes. The changes in the internal energy when the system changes from one state to another depends only on the initial and final states of the system. Internal energy is denoted by the symbol U. We can summarise the above by stating that in an adiabatic process the mechanical work (—W) done on the system

increases the internal energy of the system from U_i to U_f such that

$$-W = U_f - U_i \qquad \ldots(11.3)$$

Internal energy of a system changes during certain thermal processes i.e. during change of

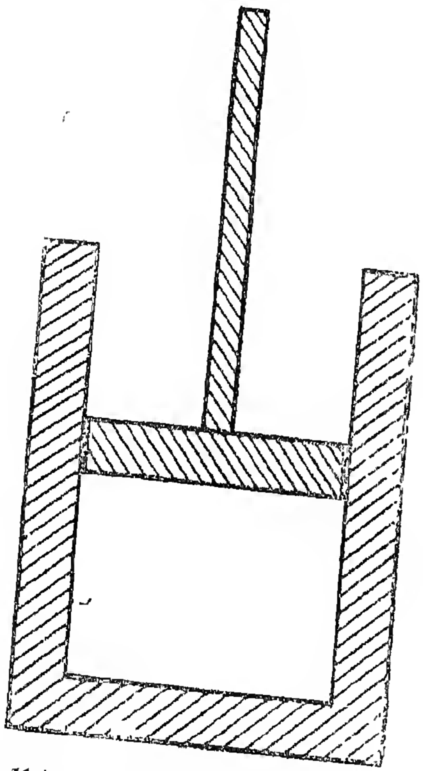


Fig. 11.4 Apparatus for adiabatic experiment

phase, say from liquid to vapour. We will discuss it while studying first law of thermodynamics.

In the above some quantities commonly used in thermodynamics have been briefly introduced. Besides, we learnt that work can be converted into heat and that a definite relation exists between the two. We will now proceed to discuss the laws of conversion of heat into work.

11.6 First Law of Thermodynamics

Consider a gas inside a cylinder with a movable piston. Let all the walls of the cylinder except the bottom be thermally insulated. Let the bottom be brought into contact with a hot body or a burner (Fig 11.5) so that it absorbs a quantity of heat Q

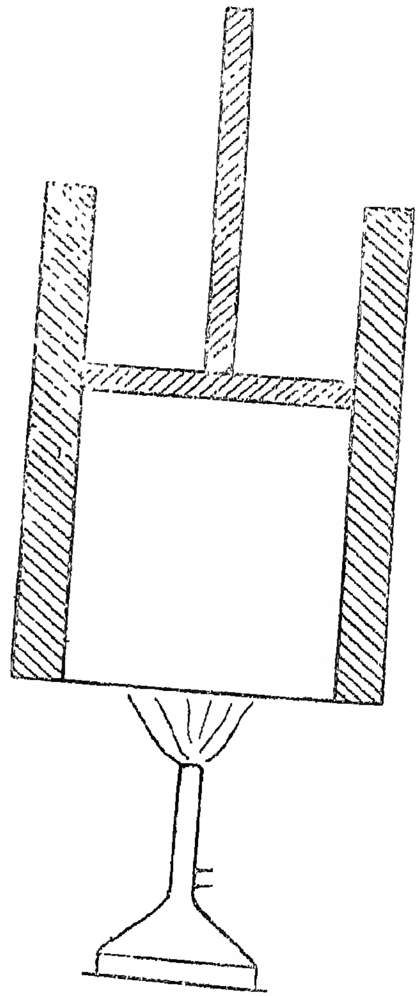


Fig. 11.5 Absorption of heat by a gas enclosed in a cylinder

On absorption of heat the temperature of the gas will rise and it will expand. In the process the piston will be pushed upwards and in doing so the gas does some work (W), and the internal energy of the gas increases from U_i to U_f.

The amount of heat energy absorbed by the gas (+Q) is used by it in doing external work (+W) and increasing its internal energy by (U_f-U_i). Since the total energy has to be conserved we can write

$$Q=W+(U_f-U_i) \quad .. (11.4)$$

The above equtaion mathematically represents what is known as the first law of thermodynamics. It can be stated in words as follows. If heat is supplied to a system which is capable of doing work then the quantity of heat absorbed by the system will be equal to the sum of the external work done by the system and the increase in its internal energy.

11.7 Applications of the First Law of Thermodynamics

(a) Boiling process

Let us apply the first law of thermodynamics to the boiling process. We know that on adding heat a substance changes its phase from liquid to vapour. For example, on being heated, water boils at 373K at atmospheric pressure.

Consider the vaporization of a mass m of a liquid at constant temperature and pressure, P. Let V_i be the volume of the liquid and V_f that of the vapour. The work done (W) by the liquid in expanding will be given by $W=P(V_f-V_i)$.

Let the heat of vapourization be L. It represents the heat needed per unit mass to change from liquid to vapour phase at constant pressure and temperature. The heat absorbed Q by the liquid will be given by $Q=mL$

By applying first law of thermodynamics to the process, we get

$$Q=U_f-U_i+W$$

or $\quad mL=U_f-U_i+P(V_f-V_i)$

Knowing m, L, P, V_f and V_i gain in internal energy can be easily calculated.

(b) Specific heat relation

While studying the Unit on Kinetic Theory of Gases we discussed the equation $C_p-C_v=R$. We will now see how this equation can be obtained by thermodynamic considerations.

The specific heat of a substance is defined as the heat per unit mass required to raise its temperature by one degree. Accordingly, the specific heat of one mole of a gas is called molar specific heat C.

Consider n moles of an ideal gas. If its temperature is raised by an amount dT at constant volume then the heat transferred is nC_vdT where C_v is the molar specific heat at constant volume. Since there is no change in the volume the work done in the process will be zero.

By applying first law of thermodynamics

$$dQ=dU+PdV$$

We get $nC_vdT=dU$

Next, instead of keeping the volume constant let the same gas be heated by keeping the pressure constant. The heat required to change its temperature by the same amount will now be nC_pdT, where C_p is the molar specific heat at constant pressure. The work done in expansion will be PdV. By applying the first law to the process we get

$$nC_pdT=dU+PdV.$$

We can assume that the change more in the internal energy of an ideal gas is entirely due to change in its kinetic energy. We have learnt in the Unit on Kinetic Theory of Gases that the change in the kinetic energy of an ideal gas depends only on the change in the temperature.

In the above, since the change in temperature dT is the same, dU will also be the same in both the cases. Substracting one from the other, we get

$$nC_pdT - nC_vdT = PdV$$

Since for an ideal gas $pV = nRT$, we can write $PdV = nRdT$. So we have

$$n(C_p - C_v)\,dT = nRdT$$

or $C_p - C_v = R$

11.8 Conversion of Heat into Work—Heat Engine

Consider the familiar case in which steam is used to do work. The working substance is steam and the heat engine i e. the mechanical device with the help of which steam undergoes a cycle, is the steam-engine. The heat is taken from the boiler and part of it rejected to the surrounding atmosphere.

In general a system which is used to convert heat into work absorbs a quantity of heat Q_1, performs an amount of work W and returns to the initial state after rejecting some heat Q_2 (Fig. 11.6). The reservoir of heat from which it absorbs heat is at a higher temperature and is called the source. The reservoir of heat to which it rejects heat is at a lower temperature and is called the sink. The magnitude of the source and the sink is so large that their temperature remain unchanged because of any heat supplied to them or removed from them.

While discussing about the work done by an engine it is useful to consider the work done in each cycle. In thermodynamics by a cycle we mean an operation satisfying the following conditions. The system starting from an initial state is brought back to the same state after undergoing through a series of processes. The system does not retrace its path and its indicator diagram is a closed curve and not a line. All the properties of the system remain unchanged during the process.

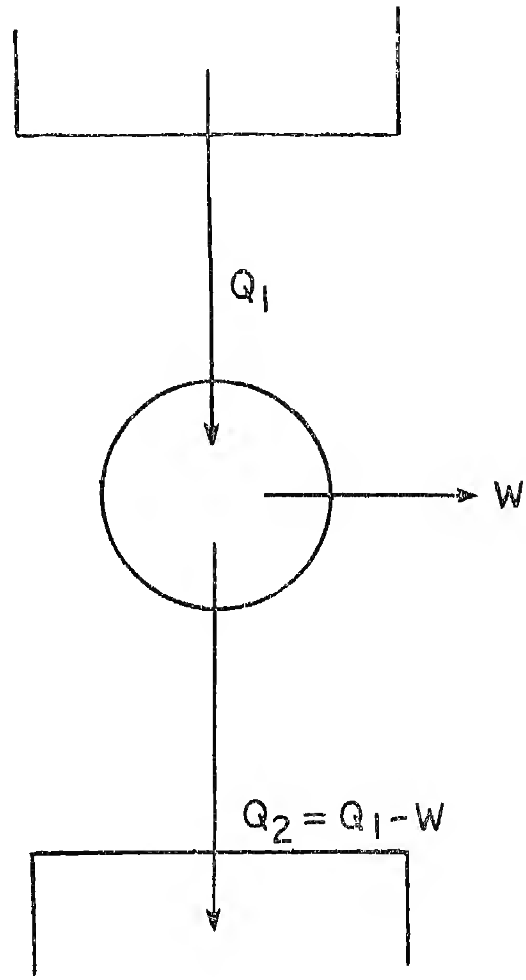


Fig. 11.6 Heat engine

The work done by a heat engine will be larger if more and more work is done in each cycle of the engine. More precisely we say that the work done by an engine is larger if its thermal efficiency is higher. Thermal efficiency

of an engine, is defined as

$$\text{efficiency} \quad (\eta) = \frac{\text{work output (W)}}{\text{heat input } (Q_1)}$$

$$\eta = \frac{W}{Q_1} \qquad \ldots(11\ 5)$$

We can substitute for W using the first law of thermodynamics Since the system returns to its initial state after completing the cycle there is no change in its internal energy So we can equate the change in the heat energy of the system to the amount of work done by the system.

$$\text{or} \quad Q_1 - Q_2 = W$$

So we can write $\eta = \frac{Q_1 - Q_2}{Q_1}$

$$\text{or} \quad \eta = 1 - \frac{Q_2}{Q_1} \qquad \ldots(11.6)$$

The value of η is found to be always less than unity. Why is it so ? This we will discuss below in our study of second law of thermodynamics.

11.9 Second Law of Thermodynamics

We have seen that heat flows from a body at higher temperature to a body at lower temperature. But it is common knowledge that heat does not flow from a body at a lower temperature to the one at a higher temperature. Why ?

We have seen that when work is done in rotating a paddle kept in a beaker containing water, the water gets heated. But why is that when a paddle is kept in a beaker containing hot water no mechanical work is done ?

We know that heat engine converts heat into work But why is it that the efficiency of an engine is always less than unity ?

All these questions lead us to second law of thermodynamics

We know that if an engine is to work continuously and more and more work has to be done, either the working substance should be inexhaustable or, it should work in a cycle. Since the first possibility is not practicable we will have to consider an engine working in a cycle In practice it is seen that no engine ever developed working in a cycle extracts heat from a hotter body and converts all of it into work without rejecting some heat to a sink at lower temperature. All of the heat extracted from the source has never been converted into work. These observations were generalised by Kelvin and also by Planck. Their statements can be combined into one equivalent statement, as "It is impossible to construct an engine, operating in a cycle, which will produce no effect other than extracting heat from a reservoir and performing an equivalant amount of work." This statement is known as Kelvin-Planck statement of the second law of thermodynamics

The above law implies that the working substance working in a cycle cannot convert all of the heat extracted into work. It has to reject some amount of heat to the sink. So in order to convert heat into work it is essential to have both a source and a sink Since all of heat extracted is never converted into work it follows that the efficiency of an engine is never unity. Of course η can never be more than one because this implies that Q_2 is negative i.e. the working substance extracts heat both from the source and the sink and rejects no heat, which we know by second law, is absurd.

11.10 Second Law and Refrigerator

We saw that in a heat engine the working substance, working in a cycle, absorbs some heat energy from a hot reservoir, converts part of it into work and transfers the rest to the sink. In practice, it is possible to construct a device which acts in the reverse way In this the working substance absorbs some heat from a sink at low temperature and on some work being done on it a larger amount of heat

is rejected to the reservoir at higher temperature Such a device is called a refrigerator, and its working substance is called the refriger-ant. The working of a refrigerator can be briefly described as follows

Let the quantity of heat absorbed by the refrigerant at a lower temperature be Q_2 (Fig. 11 7). The work done on the refrigerant be W.

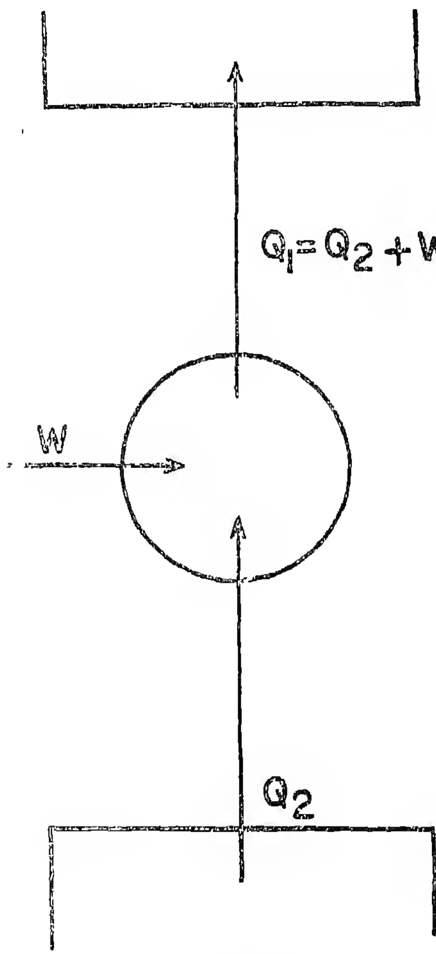


Fig. 11.7 Refrigerator

The heat rejected by the refrigerant be Q_1. The substance is working in a cycle such that there is no change in its internal energy Substituting these quantities in the mathematical form of the first law equation (Eq. 11 4), we get

$$Q_2 - Q_1 = -W$$

or $$Q_1 = Q_2 + W \qquad (11.7)$$

This equation implies that it is always neces-sary to do work on the substance in order to transfer heat from a sink to the source

In household refrigerators the work is done by an electric motor The working substance commonly used in it is freon It absorbs heat from materials kept inside the refrigerator and rejects it to the surrounding air which is at a higher temperature

11.11 Reversible Process

In thermodynamics a process is said to be reversible if it can be retraced in the opposite direction such that the system and the surro-undings pass through exactly the same states at each stage as in the direct process. After the conclusion of a process, the system and the surroundings are restored to their initial state without producing any change in either of them A reversible process has to be done very slowly such that it satisfies the following requirements at every stage of the process.

(a) The system is in mechanical equili-brium, i e there is no unbalanced force in its interior or between the system and the surroundings.

(b) The system is in thermal equilibrium, i.e all parts of the system and the surroundings are at the same tempe-rature.

(c) The system is in chemical equilibrium, i e. it does not spontaneously change the internal structure due to chemical reaction, diffusion, etc.

Since any system which satisfies the above three conditions is said to be in thermodyna-

mic equilibrium the above conditions may be summarised as follows. A reversible process at every stage during the process should be in thermodynamic equilibrium. Besides of course a reversible process should be devoid of any dissipative effects such as frictional losses, etc.

Let us consider some common processes. Motion of a body in the ground is irreversible because the energy spent in overcoming friction is not recoverable. Similarly stirring of a liquid and conduction of heat are irreversible as some energy is lost due to radiation. We can generalise that most natural processes are irreversible. All chemical reactions are irreversible because they involve change in the internal structure of the constituents. The constituents cannot be restored unless there is energy transfer from outside.

11.12 Carnot Engine

We have learnt before that the efficiency of a heat engine is always less than one. This leads us to the question as to how to operate an engine so that its efficiency is maximum. Let us try some possible solutions. Suppose we have a heat engine which works isothermally and that the working substance is an ideal gas Then on absorbing heat from the source, the gas will expand to do some work. In order to do more work, the size of the engine will have to be continuously increased and this is not practicable. Alternatively the engine must work in a cycle. But when the working substance is brought back to the initial state by doing external work the indicator diagram for the complete cycle will just be a line. Area enclosed in the curve will be zero and so no net work will be done in the process.

Next, let us suppose that the heat engine works adiabatically. On compression the temperature of the gas will rise. The compressed gas will do some work. If the gas is brought back to its original state along an adiabatic then $dQ=0$, $dU=0$ and from the first law of thermodynamics it follows that no net work will be done in the process.

So the question remains as to how to operate the engine in an efficient way. Sadi Carnot, a French engineer, worked on the idea that if the indicator diagram of an engine is made up of suitable combination of isothermals and adiabatics then more and more work can be done. He described how to carry out a set of operations so as to achieve the maximum possible efficiency in a heat engine. These set of processes constitute the Carnot Cycle. Any engine which operates by performing these processes is called a Carnot engine. Its working is briefly described below (Fig 11.8).

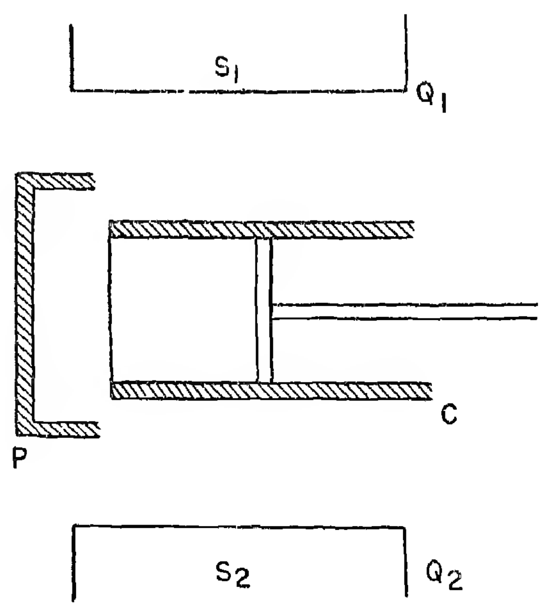


Fig. 11.8 Carnot engine

Let S_1 be the source at temperature Q_1, S_2 the sink at temperature Q_2, C be the cylinder of the engine fitted with a non-conducting

piston and P be a non-conducting cap which can be screwed to the bottom of the cylinder. Let all the sides of the cylinder, excepting its bottom be non-conducting.

Let the working substance inside the cylinder be a perfect gas The behaviour of the working substance during the four successive stages of the operation is described in Fig. 11.9.

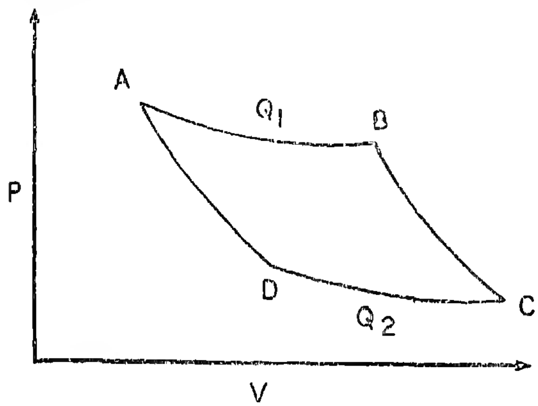


Fig. 11.9 Carnot cycle

In the first stage, the insulating cap is removed from the bottom of the cylinder The gas is kept in thermal contact with the source at temperature Q_1. The initial pressure and volume of the gas are represented by the point A in the indicator diagram. Let the gas be allowed to expand such that the piston moves very slowly, the gas expands steadily and the process is reversible. As the gas expands its temperature tends to fall. But since it is kept in thermal contact with S_1, it will absorb a quantity of heat Q_1. So all throughout the process of expansion the temperature of the gas will remain θ_1 Incidentally any process in which the temperature remains the same throughout is called an isothermal* process. During this isothermal expansion let the representative point move from A to B

At this stage when the representative point is at B the second stage of the operation begins. The insulating cap P be fixed on the bottom of the cylinder such that there is no heat transfer from the gas. But the piston will continue to move through some distance because of inertia. This motion is reversible. All through the process of adiabatic expansion due to inertial motion, there is no transfer of heat, to or from the gas. During an adiabatic expansion, temperature will fall and let the temperature of the gas at the end of the expansion be θ_2, the temperature of the sink. Let the representative point move during this adiabatic process from B to C.

At the point C the pressure of the gas is quite diminished. In order to enable the gas to do more work, it is necessary to bring it back to the original state represented by the point A This compression is done very slowly during the third and the fourth stages.

During the third stage of the process, the cap P is removed from the cylinder and the gas is kept in thermal contact with the sink at temperature θ_2 The gas is compressed isothermally Though during compression the temperature of the gas will tend to rise, it will remain constant at θ_2 because heat developed is transferred to the sink Let the quantity of heat transferred to the sink be Q_2. During this isothermal compression the representative point will move from C to D.

At this point the fourth stage of the operation begins. The cap P is replaced on the bottom of the cylinder. The gas is then further compressed in a reversible process. Since the

* iso equal, thermo heat, isotherm is the locus of points representing states of a system which are in thermal equilibrium with one state of some other system or isotherm is a locus of points having equal temperature; isothermal-pertaining to isotherms.

process is adiabatic, temperature of the gas will rise The gas is compressed till the original state is reached, i.e. till the representative point reaches the initial point A. Thus the cycle is completed. The work done by the engine during the cycle is given by the area enclosed by the curve. The gas can be taken through the cycle again and again and more and more work can be done

11.13 Efficiency of a Carnot Engine

The work done by the gas during the cycle will be equal to the area under the closed curve ABCDA It follows from the first law of thermodynamics that since the system is brought back to the initial stage, there is no change in its internal energy So this work done, W will be equal to Q_1-Q_2, i e. the difference between the heat supplied to the system and the heat rejected by the system So we can write

$$W=Q_1-Q_2$$

∴ the efficiency of the Carnot engine will be

$$\eta=\frac{\text{work done}}{\text{heat supplied}}=\frac{W}{Q_1}$$

$$=\frac{Q_1-Q_2}{Q_1}$$

$$=1-\frac{Q_2}{Q_1}$$

By calculating the values of Q_1 and Q_2, it can be shown that

$$\frac{Q_2}{Q_1}=\frac{\theta_2}{\theta_1} \qquad (11.8)$$

Where θ_1 and θ_2 are temperatures on the perfect gas scale.

So we can write

$$\eta=1-\frac{\theta_2}{\theta_1} \qquad ..(11.9)$$

11.14 Reversibility of Carnot Engine

While describing the Carnot engine we considered four reversible operations AB, BC, CD and DA. Let us now discuss the reverse process, AD, DC, CB and BA To start with let the working substance represented by the point A be allowed to expand along the adiabatic AD. During this reversible process its emperature will fall from θ_1 to θ_2

When the representative point is at D the cap is removed and the gas is allowed to expand along the isothermal DC in a reversible process by keeping it in contact with the sink at θ_2. The gas absorbs a quantity of heat θ_2 and maintains its temperature θ_2

When the representative point reaches C the cap is inserted and the gas is compressed along the adiabatic CB. During this reversible process the temperature of the gas will rise to θ_1.

When the representative point reaches B the cap is removed, the gas is kept in thermal contact with source and is further compressed along BA. In this reversible process, the gas rejects a quantity of heat Q_1 to the source.

In the above set of processes the engine takes heat (Q_2) from the sink and on some work (W) being done on it, it rejects heat (Q_1) to the source at higher temperature. The engine, therefore, acts like a refrigerator (Art. 11.10). The system and the surrounding have returned to the original state A So it follows that Carnot engine is a reversible engine.

It can also be shown that no engine is more efficient than a Carnot engine and that efficiency of all reversible engines is the same as that of Carnot engine. In the above we have considered perfect gas as the working substance. However, it can be shown that efficiency of a reversible engine is independent of the nature of the working substance.

All engines used in practice such as steam

engine, diesel engine, etc., are irreversible and their efficiency is less than that of a Carnot engine

11.15 Radiation

We know that there are three principal modes of transfer of heat energy, namely, conduction, convection and radiation. Conduction of heat through a material is caused by the transfer of heat from particle to particle and does not involve any motion of the molecules of the material itself In convection heat is propagated through the material by the actual motion of the molecules of the material In radiation heat is transferred from one body to the other even in the absence of any medium in the intervening space

Radiation is only a means of transfer of energy by transverse electromagnetic waves. It is similar in nature to light waves, radio waves, etc But in this unit we are mainly concerned with thermal radiation i.e. transfer of energy by electromagnetic waves in the infra-red region

11.16 Emission and Absorption of Radiation

Radiation emitted by a body depends on the nature of its surface A dull blackened surface is a better emitter of radiation than a silvered polished surface. Radiation emitted is directly proportional to the size of the surface A body at higher temperature will emit more radiation than that at a lower temperature. The energy emitted by a body can be expressed in terms of the emissive power of the surface

For a given temperature of a body if e_λ is the emissive power of the surface then the radiant energy emitted per square metre per second between wavelengths λ and $\lambda+d\lambda$ will be $e_\lambda d\lambda$.

By experience we know that a bright polished surface reflects most of the radiation falling on it and a rough black surface absorbs most of the radiation falling on it Different surfaces absorb radiant energy differently We define the absorptive power a_λ of a surface for a given temperature and wavelength λ as the ratio of the radiation absorbed to the radiation incident on it between λ and $\lambda+d\lambda$

11.17 Black-body

If a body absorbs all the radiation falling on it without reflecting or transmitting any of it, then it is said to be black In other words the absorptive power of a black-body is unity.

It is impossible to get a perfectly black body but some devices can be made to serve as perfectly black bodies. For example, a cavity in the form of a hollow sphere with its inside coated with black material and a small conical opening (Fig 11 10), may be considered as a

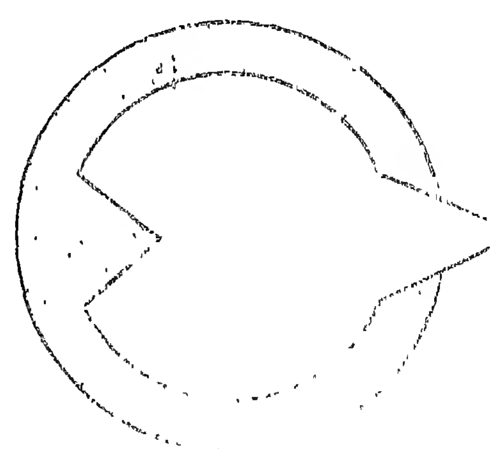


Fig. 11.10 A black-body

perfectly black body It absorbs almost all the radiation falling inside it and we cannot

see what is inside the cavity. The intensity of the radiation coming out of the hole depends only on the temperature of the cavity.

The black body commonly used in practice consists of a metallic chamber made of brass or platinum and blackened inside. It is electrically heated using heating element. The radiation coming out of a small hole in the chamber is used for experimental purposes.

11.18 Kirchoff's Law

Consider a body which is in thermal equilibrium with the surroundings. It emits some radiation as well as absorbs some radiation falling on it. Since its temperature is supposed to be constant, its rate of emission must be equal to rate of absorption. For the given temperature of the body if e_λ is the emissive power of the surface then the energy emitted per square metre, per second, between λ and $\lambda+d\lambda$ will be $e_\lambda\, d\lambda$.

Let the energy falling on unit area per second within the same wavelength range and temperature be dQ. If for the given temperature the absorptive power is a_λ then the total energy absorbed per unit area per second will be $a_\lambda\, dQ$.

Since it is assumed that the body is in thermal equilibrium, we can write

$$e_\lambda \; d\lambda = a_\lambda \; dQ$$

or $$\frac{e_\lambda}{a_\lambda} = \frac{dQ}{d_\lambda}$$

We know that for any given temperature, dQ is a constant. So for a fixed range of wavelength $d\lambda$, dQ will be a constant. This implies that $\frac{e_\lambda}{a_\lambda}$ is a constant. This is true for any surface. If we consider a perfectly black surface then

a=1 and its emissive power will be E_λ. So we can write

$$\frac{e_\lambda}{a_\lambda} = \frac{E_\lambda}{1} = E_\lambda \qquad ...(11.10)$$

We can generalise this equation as follows. "For any given temperature and wavelength the ratio of the emissive power to absorptive power is the same for all substances and is equal to the emissive power of a perfectly black body at that temperature". This law was first deduced by Kirchoff and is therefore known by his name. If the rediation covers a wide range of wavelengths then this law holds good for each wavelength considered, separately. This law implies that if a body is a good absorber of a particular wavelength of light then it should also be a good emitter of that wavelength. This law has many practical applications.

11.19 Applications of Kirchoff's Law

If a polished metal piece with a black spot on it is heated to about 1200K and then suddenly transferred to a dark room the black spot will appear brighter than the polished surface. This is because the black spot being a better absorber is also a better emitter. Similarly, when a polished green glass plate which absorbs more of red component of the light when heated to a high temperature in a furnace and taken out glows with a red light.

Kirchoff's law is of great help to study the atmosphere of the sun. Its role is illustrated by the following experiment. When a white light is viewed through a sodium flame using a spectroscope in the laboratory two dark lines are observed in the continuous spectrum. Next if a flame tinged with NaCl is observed in the spectroscope two yellow lines are seen exactly in the same position where dark lines were observed before. This phenomenon is used to explain the dark lines in sun's spectrum.

The central body of the sun consists of

glowing mass which emits a continuous spectrum without any dark lines. When this light passes through sun's atmosphere which is at a lower temperature, many elements present in it absorb their characteristic wavelengths and dark lines will be seen in those regions of the spectrum These dark lines were first studied by Fraunhoffer and are therefore known by his name. They are characteristic of different elements present in sun's atmosphere. By studying emission spectra of different elements in the laboratory and comparing them with sun's spectrum, the elements in the sun's atmosphere can be identified.

11.20 Stefan's Law

The total energy emitted by a black body depends only on its temperature The exact relation between the two was deduced by Stefan. He stated that the energy emitted per square metre of a black body per second is proportional to the fourth power of the absolute temperature, i.e.

$$E = \sigma T^4 \qquad \text{...(11.11)}$$

where σ is a constant, called Stefan's constant and has the value 5.735×10^8 J/m^2/sec/deg^4. In order to calculate the net energy lost by a hot body the above law can be generalised as follows. "If a black body at absolute temperature T is surrounded by anothe black body at absolute temperature T_o the amount of energy E_{net} lost per second per square metre by the body at higher temperature is given by

$$E_{net} = \sigma(T^4 - T_o^4) \qquad \text{...(11.12)}$$

The theoretical proof of the above equation (11.11)) was later given by Boltzmann and so the law is also called as Stefan-Boltzmann law. This law is commonly used in estimating the temperature of hot bodies as described in the article on pyrometers.

11.21 Experimental Study of Black-body Radiation

The distribution of energy of the radiation emitted by a black body among the different wavelengths of the spectrum has been experimentally studied The details of such measurements carried out by Lummer and Pringshein are briefly described here.

The source of black body radiation is an electrically heated chamber The temperature of the source is measured using a thermo-couple. The radiation is dispersed into a spectrum by using a fluorite prism. The wavelength at different regions of the spectrum can be calculated by the known formula for dispersion. The intensity of spectrum at different regions is measured using a sensitive platinum resistance thermometer called the bolometer. The electrical resistance of the resistor in the bolometer changes with the temperature, and indicates the emissive power e_λ of the source between the wave length λ and $\lambda + d\lambda$.

The distribution of energy of the spectrum for different temperatures in the range 723 K to 1046 K are shown by the curves in Fig. 11 11. Here the wavelength in microns is plotted along the abscissa and the emissive power E_λ along the ordinate.

A study of the curves clearly indicate two facts. The first indication is that E_λ increases with temperature for all wavelengths. The second indication is that each curve has a definite maximum, E_m and λ_m shifts towards smaller wavelengths as temperature increases.

The above findings support some interesting experimental observations. They justify why when a body is being heated its colour changes from red to white. They also support Stefan's law. The area enclosed between the curve and the abscissa represents the total radiation emitted by the body at a particular temperature T. By making necessary measurements it can be

seen that the area varies at T^4, and this is Stefan's law.

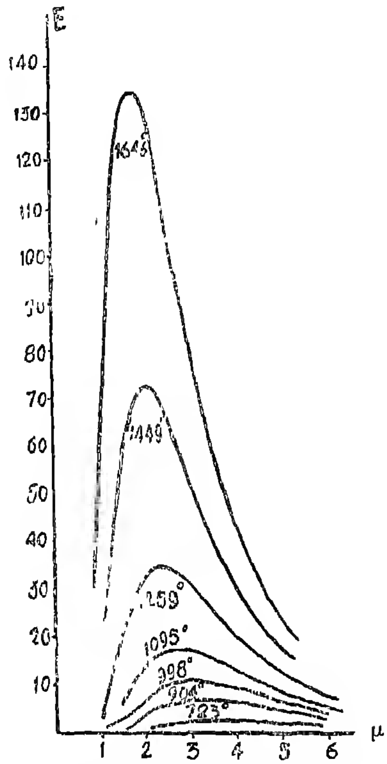


Fig 11.11 Energy distribution in a black-body spectrum

11 22 Wien's Displacements Law

It was observed from Fig. 11.11 that λ_m shifts towards lower wavelengths as the temperature is increased This observation can be quantitatively expressed by the equation.

$$\lambda_m T = \text{constant} \quad . \quad (11.13)$$

Wien derived this equation on the basis of thermodynamic considerations and hence this is known as Wien's displacement law.

The constant is found to have the value

28.84×10^{-6} n-deg.

Wien's law can be used to determine the temperature of the sun and stars. For example, investigations have shown that λ_{max} for the sun is 4753 A° On substituting this value in Wien's law the temperature of the sun comes out to be 6050 K. This value is slightly higher than the currently accepted value and this is because of the inherent assumption while using Wien's law that sun is a black body.

11.23 Pyrometers

Laws of radiations such as Stefan's law and Wien's law can be used to estimate temperature of hot bodies But the temperature thus calculated will only be approximate because these laws are applicable strictly to black bodies and the actual hot bodies are seldom black.

Instruments developed to measure high temperatures using emitted radiation are called pyrometers. These differ from other instruments such as thermometers, thermocouples, etc in that they are not to be in contact with the hot body They can measure temperature howsoever high and the lower practical limit for radiation pyrometers is about 900K. We will now briefly describe how to measure high temperatures using a pyrometer.

A typical radiation pyrometer is shown in Fig. 11.12. The beam of radiation from the hot

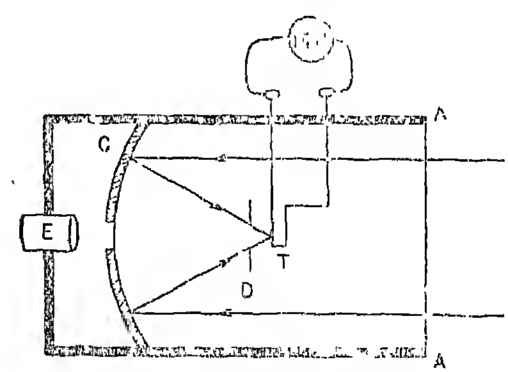


Fig. 11.12 A radiation pyrometer

body enters the instrument through the aperture AA. The concave mirror converges the beam. This focussed beam is made to pass through a limiting diaphagram D before it is incident on the thermocouple. The E.M.F of the thermocouple is recorded on a millivoltmeter connected to the terminals of the thermocouple.

The E.M.F developed is proportional to the temperature difference between the hot and the cold junctions. If T and T_0 are the temperatures of the hot body and the receiving junction, respectively, then the E.M F. developed should be proportional to $T^4 - T_0^4$. Since T is very large as compared to T_0, T_0^4 can be neglected in comparision to T^4.

In practice it is found that E.M.F. is not exactly proportional to the fourth power of the absolute temperature of the source. Some of the reasons for this discrepancy are : (i) the source is not a black body, (ii) T_0^4 is not zero, and (iii) stray radiations and conduction of heat raise the temperature of the cold junction. So, before use, the pyrometer has to be suitably calibrated, and it is found that the value of the power varies from 3.8 to 4.2.

Exercises

11.1 Distinguish clearly between temperature and heat.

11.2 In a joule experiment two weights of 5 kg each fall through a height of 3m and rotate a paddle wheel which stirs 0 1 kg water. What is the change in the temperature of the water ? (0.7K)

11.3 A lead bullet weighing 20g and moving with a velocity of 100m/s comes to rest in a fixed block of wood. Calculate the heat developed and the rise in temperature of the bullet assuming that half the heat is absorbed by the bullet. The specific heat of lead is 0.03. (23.8 cal, 19.9K)

11.4 1g of water at 373K is converted into steam at the same temperature. The volume of 1 cc of water becomes 1671 cc on boiling. Calculate the change in the internal energy of the system if the heat of vaporisation is 540 cal/g. (499 cal)

11.5 Apply the first law of thermodynamics to obtain an expression for the change in the internal energy during the melting process

11.6 Discuss whether the following phenomena are reversible :
   A. Waterfall
   B. Rusting of iron
   C. Electrolysis.

11.7 Give two examples of a reversible process. Discuss their reversibility.

11.8 What is meant by a reversible engine ? Explain why the efficiency of a reversible engine is the maximum.

11.9 A steam engine takes steam from the boiler at 500 K and rejects it to the air at 373K. What is its efficiency ? (25.4%)

11.10 In a refrigerator heat from inside at 277K is transferred to a room at 300K. How many joules of heat will be delivered to the room for each joule of electric energy consumed ideally ? (13 joules)

11.11 What is meant by the emissive power of a surface ? Explain why the radiation in a cavity depends only on the temperature of the walls and not on the material of which it is made ?

11.12 Explain how Kirchoff's law helps to identify the elements in the sun's atmosphere. Why does a piece of red glass, when heated and taken out, glow with a green light ?

11.13 Wavelength corresponding to E_{max} for the moon is 14 microns. Estimate the temperature of the moon by taking the value of σ given in the book. (200K)

11.14 Describe a method to estimate the temperature of a hot furnace.

11.15 Using Stefan's law deduce Newton's law of cooling which states that "If the temperature difference between a hot body and its surroundings is not large then the rate of cooling is proportional to the excess of temperature of the substance above the surroundings."

UNIT 12

Liquids

12.1 Intermolecular Interactions

In kinetic theory of gases, we have seen that matter is composed of minute particles called molecules It is assumed that there are no forces acting between them. In other words, the intermolecular interactions are supposed not to exist. But, in fact, these are simplifying assumptions. Even for gases, molecules have finite size and they interact with one another. It is for these reasons that the equation of state for a real gas differs from that for the ideal gas PV=RT. Van der Waals modified the equation of state of an ideal gas by making a correction in the pressure*. The reason for this correction is the presence of intermolecular interaction in real gases. This intermolecular interaction is therefore known as Van der Waals' interaction. It exists not only in gases but also in liquids and even in some solids (Art 16 6) There are three general categories into which Van der Waals' interaction can be divided :

(1) Interaction due to dipole-dipole forces.
(2) Interaction due to induced dipole forces (also called induction forces).
(3) Interaction due to dispersion forces.

Van der Waals' interactions (each one of the above three) vary as $\frac{1}{r^6}$ where r is the intermolecular separation between the molecules. The dipole-dipole interaction is the strongest of the three and the dispersion interaction the weakest Evidently the respective forces vary as $\frac{1}{r^7}$ with intermolecular separation.

Before proceeding further it will be worthwhile to have a look at the origin of the interactions Even though the net charge on the molecule is zero, it is composed of positive and negative charges. In spite of the electrical neutrality of the molecule, however, the distribution of positive and negative charges on it is not even. The atoms in the molecule are so arranged that the centre of mass of the positive charges does not fall exactly on the centre of mass of the negative charges.

The positive and negative charges, separated by a distance, form an electric dipole. The molecular dipole has a dipole moment whose behaviour in an electric field is the same as that of a magnet in a magnetic field. Molecules having permanent dipole moment, like H_2O, are known as polar molecules, in contrast

* Van der Waals also applied a correction in volume due to the finite size of the molecules. We are here not concerned with this aspect

to non-polar molecules like CO_2, O_2, N_2, etc. Since the dipole-dipole interaction is strong, generally polar molecules are liquid under ordinary physical conditions (water, alcohol, etc.).

The nature of induction and dispersion forces is difficult to understand and will not be treated at this stage. However, the ultimate nature of induction as well as dispersion forces is also of the dipole-dipole type. The dispersion force, which is the weakest of the three, can occur in all cases and even in monoatomic gases like argon, etc.

Thus, it is seen that the molecular interactions are electric in nature. The molecular forces are (i) attractive at large distances, and (ii) have necessarily to be repulsive at very close distances. The nature of intermolecular forces has been shown in Fig 12.1. The attrac-

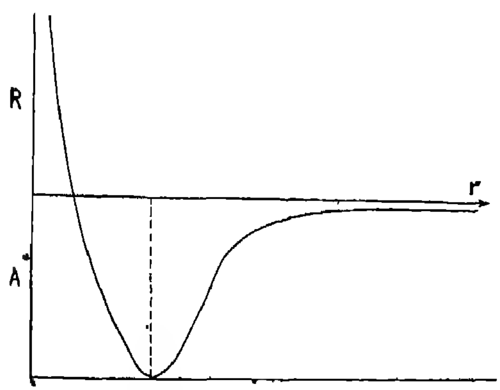


Fig. 12.1 Nature of intermolecular force
R—repulsive force A—Attractive force.
r—separation

tive force increases rapidly with decrease in separation between molecules for larger distances. Some time it is convenient to imagine a small sphere of radius of about 10^{-9}m around a molecule and call it its sphere of action Since interactions fall rapidly with distance it is useful to regard the molecule as exerting an attractive force on other molecules lying within its sphere of action and neglect its influence on others lying outside this sphere In some diagrams in this unit this sphere has been shown by a dashed circle.

When the separation becomes too small the repulsive forces start dominating even more rapidly. Had there been no repulsive forces all matter would have collapsed to a point. The intermolecular separation (r_e) maintains an equilibrium position where the potential energy due to repulsion and attraction is a minimum. These repulsive forces are also electric in origin and are due to point charges in the atoms of the molecule which form the molecule.

The minimum potential energy corresponds to $r_e \simeq 10^{-9}$ m and the repulsive force varies more sharply with decreasing intermolecular distance, approximately as $\frac{1}{r^9}$.

The combination of the intermolecular forces and thermal motion gives rise to three states of matter, i e. solid, liquid and gas. In solids the attractive forces are quite strong and thermal motions cannot break them away. Molecules may stay at one place and vibrate. In gases the intermolecular attractive forces are so weak that random thermal motion can very easily overcome them and molecules are almost free to move about anywhere Liquids fall in between these two cases. For the sake of simplicity a liquid is sometimes visualized as a dense gas. The molecules in a liquid are neither forced to stay permanently in an equilibrium position nor they are free to leave the company of other molecules. A molecule inside a liquid can slide freely over others without any effort.

Cohesive and adhesive forces

The attractive forces between similar molecules discussed above are called cohesive forces.

The forces between dissimilar molecules e g. the molecules of glass and water are called adhesive forces. The attractions are called cohesion and adhesion respectively We can thus define a liquid as a state of matter in which the molecules can change their position with respect to each other but are restricted by molecular forces or cohesive forces so as to maintain a fixed volume. It is also known that if we want to convert a liquid into vapour, we have to supply an extra amount of energy. To separate two molecules the energy supplied to them should at least be equal to the intermolecular interaction energy. Intermolecular energy in liquid molecules can be calculated from the latent heat of vaporization of the liquid.

EXAMPLE 12.1

The latent heat of vaporization for water is 22.6×10^5 Joule/kg. Calculate the intermolecular binding energy. (Avogadro Number $N = 6 \times 10^{23}$ mole^{-1}, 1 Joule$=0.62 \times 10^{19}$eV).

Solution

Molecular weight of water $= 8$

Number of molecules in 1kg of water $N = 6 \times 10^{26}/18 = 10^{26}/3$.

Energy required to unbound

N molecules of water $= 22.6 \times 10^{+5} \times 0.62 \times 10^{19}$eV

$= 1.4 \times 10^{25}$ eV.

Intermolecular binding energy

$$= \frac{1\ 4 \times 10^{25} \times 3}{10^{26}} \text{ eV}$$

$$= 0.4\text{eV}.$$

12.2 Surface Tension

A dead fly or a beetle tends to sink to the bottom when immersed in water. This indicates that its average density is larger than that of water. And yet we see in nature large water beetles running on the surface of a lake seemingly without wetting their feet. A sewing needle placed carefully on the surface of a liquid floats at the surface, though the density of the needle's materials may be as much as eight times that of water. A liquid flowing slowly from the tip of a medicine dropper does not emerge as a continuous stream but as a succession of drops. Rain drops, fog drops, soap bubbles, etc. assume spherical shapes as they fall through the air. All these phenomena, and many others of similar nature, are associated with the existence of a boundary surface between a liquid and some other substance. Let us now consider the properties of the liquid surface.

From the consideration of a molecular interaction we have seen that unlike solids, liquids do not have any definite shape of their own. Even though weakly, the molecules of a liquid interact with one another Liquids attain the shape of the container in which they are kept. Due to gravitation the surface of a liquid is plane and horizontal in a wide vessel. In the presence of adhesive forces, that is, interaction of the molecules of the material of container and molecules of the liquid the shape of the liquid surface is deformed near the walls of the container In narrow tubes, therefore, the surface of the liquid appears concave or convex. However, in wide vessels, as stated above, the distortion which is only at the boundary is negligible and the surface of the liquid is plane. The intermolecular (or cohesive) forces discussed in Art. 12 1 explain the interesting behaviour of a liquid (Fig. 12.2 a). It is surrounded by similar molecules from all the sides. The net force on such a molecule is zero. All the molecules inside the liquid are in similar approximate environment and have more or less the same potential energy. Hence, these molecules slide from one place to another inside the liquid without doing any work. The situation is different when we consider a molecule. The molecule B on the

surface is attracted by many molecules from below but there is no attractive force from above the surface (the force due to molecules

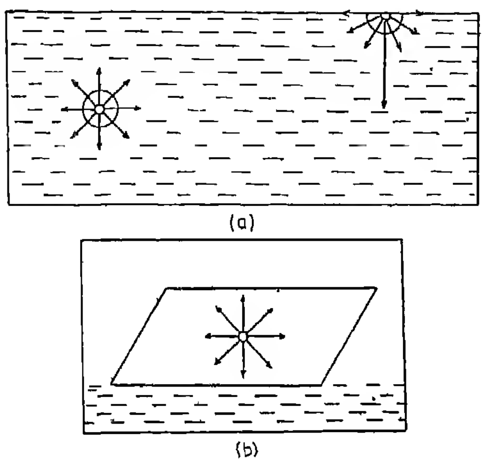


Fig. 12.2 (a) Molecule inside a liquid
(b) Molecule on the surface of a liquid

in liquid vapour can be neglected). A molecule on the surface will therefore be attracted in all directions by the molecules beneath. We can resolve all these forces in the vertically downward direction The resultant picture is that every molecule on the surface is acted on by a vertically downward pull inside the liquid. Due to this downward force, the potential energy of the molecules on the surface is larger than that of those inside. In consequence, the surface of a liquid tends to contract as much as possible. In other words, the surface of a liquid is under tension. In Fig. 12.2 (b) a molecule on the surface is shown to be acted upon by the molecular forces of the neighbouring surface molecules symmetrically in all directions. The net force is zero but the surface acts like a stretched membrane. Such a tension is measured in a stretched membrane across unit length of an imaginary straight lined rawn on the surface. The surface tension of a liquid therefore is defined as the force of contraction across an imaginary line of unit length tangential to the surface of a liquid, the line and the force being perpendicular to each other. The unit of surface tension is dyne per cm in c g s. or Newton per metre (Nm^{-1}) in MKS, system.

TABLE 12.1

Surface Tension of Some Liquids

Liquid	*In Contact with*	*Newton/metre* $\times 10^{-3}$
Olive oil	air	35
Benzene	air	29
Glycerine	air	63
Water	air	75
Mercury	air	35

As an example we see that a bug floats on water due to surface tension. The bug (Fig. 12.3) bends its legs on the surface of water such that the deformed water surface gives rise to forces of surface tension along the directions as shown in Fig 12.3. The weight of the bug

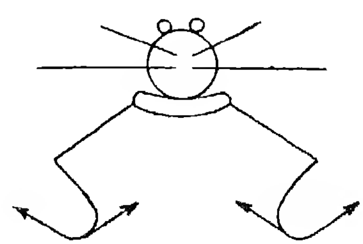


Fig. 12.3 A bug upheld on water surface by surface tension

is neutralized not by buoyancy (the bug is hardly submerged) but by the upward components of the surface tension forces which are tangential to the deformed surfaces.

As mentioned earlier, another example to understand the phenomenon of surface tension is that liquids show a tendency to assume a characteristic spherical shape. A molecule of a liquid deep inside a drop is surrounded from all sides by other molecules To remove this molecule A (Fig 12.4) from inside the

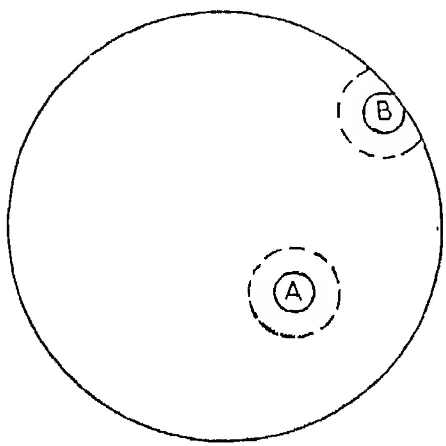


Fig. 12.4 A—molecule inside a drop
B—molecule on the surface of the drop

drop we have to do work against the attraction of all its neighbours. Since we must do some work to remove this molecule from the drop, it apparently had a quite large negative energy, say $-E_1$, when it was inside the drop. Let us repeat the process for a molecule B, on the outer surface of the drop This molecule, being on the surface, is not surrounded by neighbours on all sides except from beneath. Therefore, less work will have to be done to remove this molecule. Let the energy of the molecule B on the surface be $-E_s$. It is obvious that $|E_s| < |E_1|$ or $-E_s > -E_1$, that is, the energy at the surface is positive as compared to the energy in the interior of the drop We know that the physical systems come to a state of least energy if they are able to (a car rolls down hill, a hot body becomes cooler, and so on) reduce the number of its high energy molecules and therefore its total energy So a piece of liquid will do its best to make its surface as small as possible. The geometrical shape that has the least surface for a given volume is a sphere; so the liquid comes as close as it can to assume the form of a spherical drop. The departure from the spherical shape is due to the weight of the drop as well as the adhesive forces if any It is seen that as soon as the water drop falls on a glass plate it flattens. This is due to the adhesive force between glass molecules and water molecules.

12 3 Surface Energy

We have seen that the molecules on the surface of a liquid which are pulled inward (like weights on a shelf which are pulled earthward by gravity) possess potential energy. As a molecule enters the interior it loses energy. Thus, because a molecule of a liquid on the surface is at higher potential energy than a molecule of the liquid, deep inside, one has to do certain amount of work in creating a liquid surface In other words, if we want to take a molecule from inside the liquid to the surface, we will have to work against the downward pull Thus, if we want to create a surface, the work of increasing the number of molecules on the surface has to be done. The energy required to create a surface is known as surface energy. We can calculate this energy by means of a simple experiment. A rectangular frame of wire, ABCD, is taken. The arm AD of the frame is not fixed to the frame but can slide,

resting on the arms CD and BA. The frame is carefully dipped in a soap solution so that a film ABCD is formed. Let the sliding arm AD be of length l (Fig. 12.5) and the surface tension

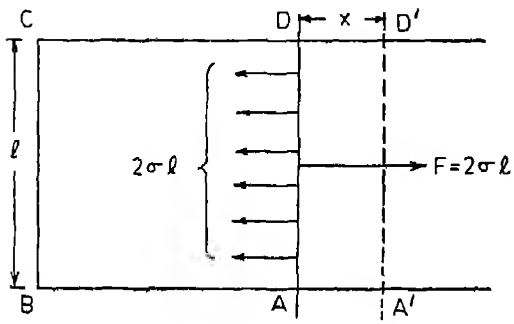


Fig. 12.5 Increasing the area of the drop

of the liquid be σ. Then by definition of surface tension, the force acting on the wire is

$F=\sigma 2l$, since the film has two surface.

If DA is slid to a new position D′A′ by a distance x, then the increase in the surface area of the film is

$\triangle S=2lx$

The work done in creating this new surface $\triangle S$ is given by

$$\begin{aligned}W&=F\,x\\&=\sigma\, 2lx\\&=\sigma\,\triangle S\end{aligned}$$

$=\sigma\times$increase in the area of the film.

Thus, the work done to increase the surface area of a film by unit amount is equal to the surface tension of the liquid. Therefore, σ is also called the free surface energy per unit area. In actual practice the force acting on wire AD (Fig. 12.6) can be measured by a spring balance. Suspend a frame of wire like ABCD from the hook of the balance. CD is fixed to the frame and is not free to slide. As

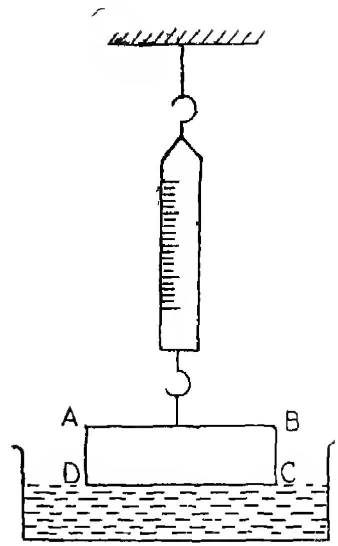


Fig. 12.6 Measuring the force due to surface tension

soon as the arm CD is brought in contact with the surface of a liquid in a beaker, the wire is pulled down. This force can be measured by the spring balance. It will be found that the force Fl which is the pull on the spring, is equal to $2\sigma l$.

EXAMPLE 12.2

When two mercury drops are brought into contact, they form one drop. Explain.

Solution

Let each of the small drops have a radius r,

Volume of each drop $=\frac{4}{3}\pi r^3$

Surface area of each drop $=4\pi r^2$

Total surface energy of the system of two drops $=8\pi r^2\sigma$

where σ=surface tension of mercury.

When both drops unite, the radius of the resulting drop is given by the equation

$$\frac{4}{3}\pi R^3 = \frac{8}{3}\pi r^3$$

where R is the radius of the resulting drop.

So $R = 2^{\frac{1}{3}} r$ and

Surface energy of single larger drop

$= 4\pi r^2 2^{\frac{2}{3}} \sigma$

Hence the surface energy of the large drop is less than the total surface energy of the smaller drops. As every physical system tries to attain a state of minimum energy, the drops of mercury coming in contact form one drop.

Conversely, if you want to break any drop of liquid into smaller drops you will have to do work.

The low value of surface tension helps the liquid to spread out as a thin film as the spreading is easier on account of low surface energy Low surface tension also allows a liquid to penetrate in narrow spaces or pores. Water has relatively high surface tension. But if detergent is added to it, the surface tension is lowered. The cleaning property of the solution is improved because now the solution can seep into fine pores where water could not.

So far we have considered only a free liquid surface. But actually a liquid surface is either in contact with a solid or vapour or some other liquid Consider a system consisting of a solid S and liquid L (Fig. 12.7). Suppose

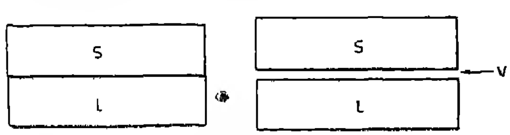


Fig. 12.7 A system containing a solid S, liquid L and vapour V

that they are initially in contact, and are then separated. Let the work per unit area required to separate them be W_{SL}. Before separation there is potential energy in the interface amounting to σ_{SL} per unit area (σ_{SL} is the surface tension of solid-liquid film). After separation there is energy σ_{SV} per unit area on the surface between the solid and vapour (σ_{SV} is the surface tension of solid-vapour film) and σ_{LV} per unit area on the surface between liquid and the vapour (σ_{LV} is the surface tension of liquid-vapour film). The initial energy of the system plus the work done on the system for separating the solid from liquid is equal to the final energy after separation. Hence we have

$$\sigma_{SL} + W_{SL} = \sigma_{SV} + \sigma_{LV} \qquad \text{... (12.1)}$$

12.4 Angle of Contact

Liquid has to be kept in a container. The molecules of container are different from those of liquid On the walls of the container three surface films viz, liquid-vapour, solid-vapour and solid-liquid are formed. The films are only a few molecules thick. Associated with each film is an appropriate surface tension, (Fig. 12.8). The curvature of the surface of liquid near a solid wall (or the shape of a liquid drop on a surface) depends upon the difference between σ_{SV} and σ_{SL}. At the wall the three films meet. If we isolate a small portion of all three films at their junction and imagine the film to extend a unit distance perpendicular to the diagram, the isolated portion will be in equilibrium under the action of four forces, three of which are surface tensions of the three films. The fourth force F_a is adhesive force between isolated portion of film and the wall. From equilibrium condition, we get

$$\sigma_{LV} \sin\theta = F_a$$

$$\sigma_{LV} \cos\theta = \sigma_{SV} - \sigma_{SL}$$

θ is the angle between σ_{LV} and σ_{SL} taken inside the liquid and it is called the angle of

contact Eliminating

$\sigma_{SV}-\sigma_{SL}$ we get

$$W_{SL}=\sigma_{LV}\,(1+\cos\theta) \qquad (12.2)$$

If the work done per unit area required to separate liquid from the solid is greater than

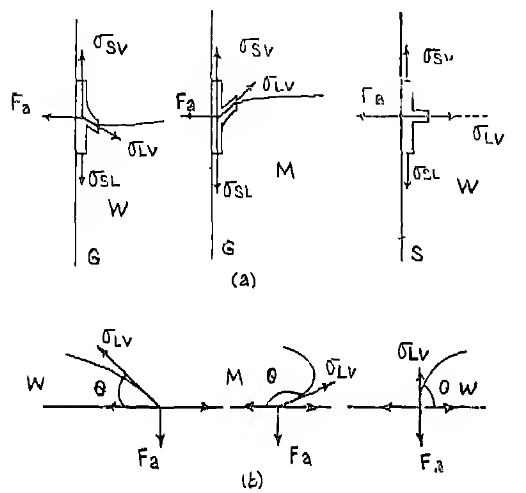


Fig. 12.8 (a) Layer of a liquid near the wall of a container
W—water
M—mercury, S—silver, G—glass
(b) Drop of a liquid on a surface

the surface tension of liquid-vapour film the angle of contact θ is acute, that when $W_{SL}>\sigma_{SV}$, $\theta<90°$ and $W_{SL}<\sigma_{LV}$ $\theta>90°$

When water is taken in a thin glass tube the meniscus of water is concave, where as the meniscus of mercury is convex This is because the work done in removing a unit area of water surface from the glass surface is more than the surface tension of water-vapour film Another way of explaining it is that in the case of water-glass-water vapour system we have $\sigma_{SV}>\sigma_{SL}$ In case of mercury-glass-mercury vapour system $\sigma_{SV}<\sigma_{SL}$, the angle of contact is obtuse. Thus the meniscus of mercury in glass is convex When the water is kept in a silver container $\sigma_{SV}\cong\sigma_{SL}$ and θ is 90°.

A solid surface will be wetted by the liquid if θ is small, but when θ is larger than 90° the liquid will slip over the surface Impurities and adulterants present in or added to a liquid may alter the angle of contact considerably Water proofing agents applied to a cloth cause the contact angle of water in contact with the cloth to be larger than 90°

12.5 Pressure Difference Across a Surface Film

When the free surface of a liquid is plane (Fig 12 9 (a) the resultant force due to surface tension on a molecule in the surface is zero. If the surface is curved, then there is a resultant force normal to the surface This resultant force is directed into the liquid in the case of convex surface (Fig. 12.9 (b). In order to keep

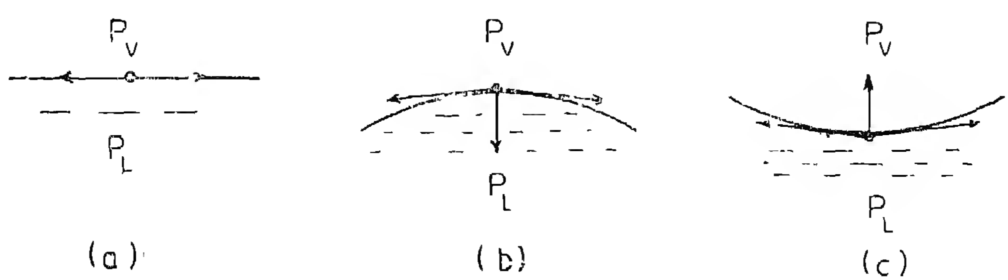


Fig. 12.9 Pressure difference across a surface film
(a) Plane surface; (b) convex surface, (c) concave surface

this surface in equilibrium the pressure on liquid side of the surface must be more than the pressure on the vapour side, is, $P_L > P_V$ Similarly, it can be shown in the case of a concave surface $P_V > P_L$. Hence, we conclude that the pressure on the concave side of the film is always greater than the pressure on the convex side.

The argument given already shows that whenever the surface of a liquid is curved there exists a difference of pressure inside and outside the surface Consider for example a spherical drop of a liquid of radius R Due to the spherical shape there will exist excess pressure inside the liquid as compared to outside atmospheric pressure. Let the excess be P. Let us enlarge the liquid drop from radius R to R+dR. This increases the surface area of the drop from S to S+dS.

Since $S=4\pi R^2$ $\quad dS=8\pi R dR$

From the definition of surface tension the work done in increasing this surface area is σdS. Since work done is also equal to force multiplied by distance and force is equal to pressure × area We have

$$P \times 4\pi R^2 \, dR = \sigma 8\pi R dR$$

$$\text{or } P = \frac{2\sigma}{R} \qquad (12\ 3)$$

It follows from this result that if the surface tension remains constant, the pressure difference is larger for smaller value of R Due to this large internal pressure, tiny fog drops have rigidity properties like those of solids A good illustration is to be found in the case with which ice-skates slide over the surface of smooth ice. Under the enormous pressure exerted on the ice by the sharp metal edge of the skate, ice melts, and the runners run along on the tiny drops as if they were on ball bearings.

EXAMPLE

Find the difference in air pressure between inside and outside of a soap bubble 5 mm in diameter. Assume the surface tension to be 1 6 N m^{-1}

Solution

Unlike a drop of liquid, the soap bubble with air inside has two surfaces, one external and another internal Due to two spherical surfaces the expression for the excess of pressure inside a soap bubble is given by

$$P_{excess} = 4\sigma/R,$$

$$= \frac{4 \times 1\ 6}{2.5 \times 10^{-3}} = 2560 \text{ N m}^{-2}$$

12 6 Capillarity

The most familiar surface effect is the elevation of a liquid in an open tube of small cross-section The term 'Capillarity', used to describe effects of this sort, originates from the description of such tubes as capillary or 'hair-like' (Fig 12 10) In case of a liquid that wets the tube, the contact angle is less than 90° and the liquid rises until an equilibrium height 'h' is reached When the contact angle is greater than 90° the liquid is depressed inside the capillary as in mercury. If the pressure on the concave side of the meniscus is P then that on the convex side is $P' = P - \frac{2\sigma}{R}$ (from equation (12 3) since we can very easily see that $P - P'$ = the pressure due to the liquid column of height 'h')

In case of a liquid forming a concave meniscus in a glass capillary : P is the atmospheric pressure, and as $P' < P$ the liquid must rise in the capillary till the weight of the water column is such that it exerts a pressure $P - P'$ at its base If the angle of contact is θ, and the radius of capillary is R, the radius of meniscus

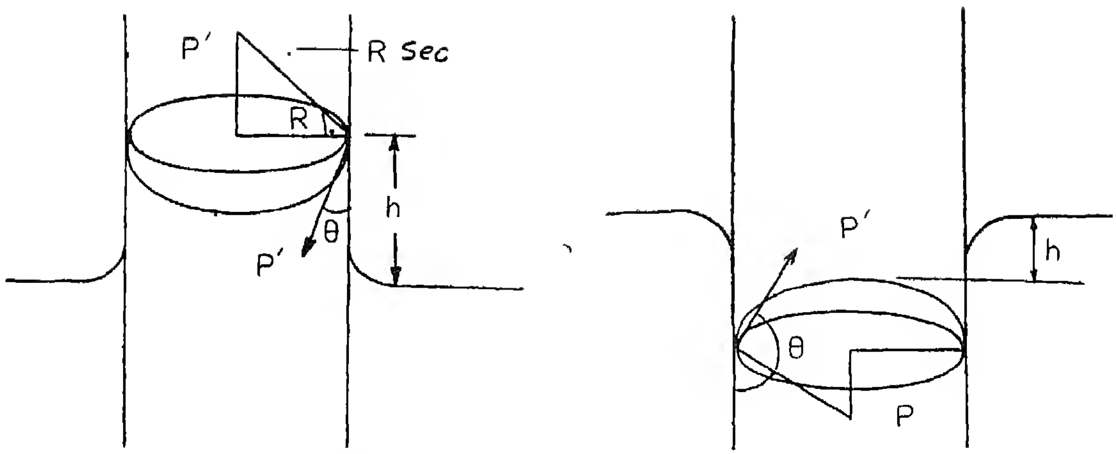


Fig. 12.10 Liquid column inside a capillary

is given by R secθ. The excess of pressure is given by

$$P-P'=2\sigma/R\sec\theta$$

Therefore $2\sigma\cos\theta/R=h\rho g$ (ρ=density of the liquid)

$$\text{or}\quad h=\frac{2\sigma\cos\theta}{R\rho g} \qquad \ldots (12.4)$$

Here we have neglected the small volume of the meniscus. In case of water, $\theta=0$,

$$h=\frac{2\sigma}{R\rho g}$$

The same equations hold for capillary depression.

We can find out the surface tension of a liquid if we know the angle of contact. The rise of liquid in capillary and the radius of capillary are measured with the help of a travelling microscope. It should be noted that before inserting the tube in the liquid, the tube must be cleaned. If there is any impurity at the inside surface of the capillary, the angle of contact will change, thus causing error in the measurement. The capillarity phenomenon is responsible for the rise of oil in a wick lamp, the absorption of ink in a blotting paper, etc. If the height of capillary is smaller than h given by equation (12.4) the water will not continue to flow out of the top of capillary as a fountain. The uppermost periphery of water will not be in contact with the walls of capillary. The actual shape of meniscus will change and the radius of curvature of meniscus will be such that the difference of pressure across the meniscus will only maintain a water column or smaller height.

12.7 Flow of Liquids

In summer the flow of water in rivers and canals seems to be steady. The same rivers when in flood show a very unsteady flow. There appear whirl pools and vortices on water surface. If a liquid in a vessel is stirred and then left to itself, the motion will disappear after some time These examples show that liquids under different conditions flow differently. In flowing liquids, it is not necessary, that at any particular instant all the molecules have the same velocity. The molecules or molecular layers can move relative to each other with different velocities. Due to relative motion between different molecular layers, frictional force acts, causing resistance to the motion of the liquid. This resistive

property is known as viscosity. Therefore, in order to slide one layer of liquid over another an external force must be exerted to provide uniform motion to a liquid.

Let now the particles of a liquid be in motion. When the liquid velocity at any given point is constant of time, the motion is said to be steady. Whichever particle passes through the point A, it has a velocity V_A in a steady flow (Fig 12.11). When one particle leaves A

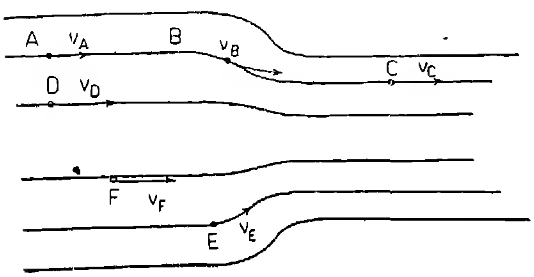


Fig. 12.11 Steady flow of a liquid

with velocity V_A, it is followed by another particle. When this second particle leaves A it also has a velocity V_A. Similar thing must occur at all other points in the liquid, that is, the liquid particle must always leave B with velocity V_B, C with velocity V_C, D with velocity V_D and so on. It is not necessary for these velocities to be equal.

In a steady flow let a liquid particle follow the path ABC. If the liquid particles preceding it and following it also follow the same path as ABC, the path is called a line of flow or a stream-line. A stream-line is parallel to the velocity of fluid particles. No two stream-lines can cross each other. The stream-line motion is possible only when the liquid velocity is low and the velocity along a path does not change too abruptly. If the velocity is high, or the change in velocity is frequent and very abrupt, the motion ceases to be a stream-line flow

In non-viscous liquids, the velocity of all the particles at one section of a pipe are equal and liquid advances as a unit along a pipe. The surface determined by heads of velocity vectors is a plane, and the fluid flow is characterized by a plane velocity profile (Fig. 12.12 a). When the fluid is viscous and the velocity is not large, the flow is called laminar. (The lamina of liquid slide over each other). The velocity profile has the shape shown in Fig. 12.12. The velocity is maximum along

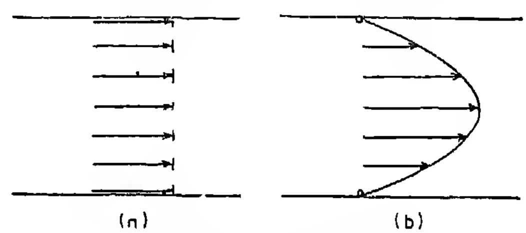


Fig. 12.12 Flow of liquid through a pipe
(a) Non-viscous liquid
(b) viscous liquid

the axis of the pipe and decreases to zero at the wall. When the velocity exceeds a certain critical value, the nature of flow becomes very much complicated. Random, irregular, local circular currents called vortices, develop throughout the fluid. The resistance to the flow increases tremendously. This type of flow is called turbulent flow.

It has been observed experimentally that there is a combination of four factors which determines the nature of flow of a viscous fluid through a pipe. This combination is known as the *Reynold number*, N_R and it is defind as

$$N_R = \frac{\rho V_D}{\eta} \qquad \dots (12.5)$$

Here ρ is the density of the liquid, v is its average speed, D is the diameter of the pipe and η is the coefficient of viscosity of the

liquid. The flow of viscous fluid is said to be laminar when N_R lies between 0 and 2000 For values of N_R above about 3000, the flow is turbulent. For N_R between 2000 and 3000 the flow is unstable and may switch over from one type to another. Reynold number is a pure number and therefore its numerical value is the same in any consistent set of units.

EXAMPLE 12 4

What should be the maximum average velocity of water in a tube of diameter 2 5 cm so that the flow is laminar ? The viscosity of water is 0 001 $Nm^{-2}S$

Solution

The maximum value of Reynold number for a laminar flow is 2000. Let V be the maximum average velocity of water for laminar flow Then

$$\frac{\rho VD}{\eta} = 2000$$

$$\text{or} \quad V = \frac{2000 \times 0.001}{10^3 \times 2.5 \times 10^{-2}} = 0.08 \ ms^{-1}$$

12 8 Viscosity

As mentioned earlier viscosity may be thought of as the internal friction of a liquid Because of viscosity, a force must be exerted to cause one layer of liquid to slide past another. If there is a layer of liquid between two sheets a force is needed to slide one sheet over another. Let a liquid flow over a fixed surface such as AB (Fig. 12.13) The layer of liquid in contact with AB remains at rest while the upper most layer PQ has the maximum velocity V The layer EF at a distance x + dx from AB flows with a greater velocity than that of a layer CD at a distance x from AB. If the difference in the velocities of these two layers is dV, the velocity gradient between layers CD and EF will be dV/dx Every lower layer tries to check the flow of the adjascent upper layer This is equivalent to the statement that the flow of the liquid is opposed by

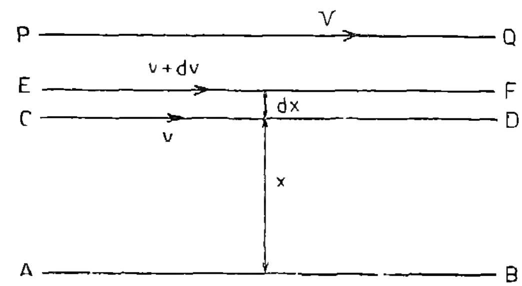


Fig. 12.13 Flow of viscous liquid over a surface

a tangential force F offered along the surface of the layer. It is found by experiment that the force per unit area offered by liquid layers is proportional to the velocity gradient
That is,

$$F/A \propto dV/dx$$

$$\text{or} \quad F = \eta A \ dV/dx \qquad \text{.. (12.6)}$$

The proportionality constant η, is called the coefficient of viscosity or simply the viscosity. Poise is defined as a tangential force per unit area offered by a liquid layer to create a unit velocity gradient The dimension of η can be easily calculated as

$$\eta = \frac{F}{A} \Big/ \frac{dV}{dx} = \frac{MLT^{-2}}{L^2} \Big/ \frac{L}{T\,L} = ML^{-1}\,T^{-1}$$

We have already seen that when a viscous liquid flows through a tube, the flow is laminar depending upon the radius of the cross-section of the tube and the average velocity of liquid. The force required to maintain a laminar flow of liquid through a tube of length l and radius r is given by

$$F = 8\eta l \ \frac{V}{r^2} \qquad \text{... (12.7)}$$

where V is the volume of the liquid flowing per second through the tube.

Stoke's Law

So far we have discussed the flow of a liquid on a solid surface or through a pipe. Let us now consider the flow of a viscous liquid past a body or the motion of a body through a viscous fluid at rest. A steel ball falling in a tank of glycerine attains a constant velocity in a very short distance from the start of the motion. This shows that the retardation due to resisting force on the steel ball is equal to the acceleration due to gravity. When a spherical body moves through a viscous liquid at rest (Fig 12.14), it is expected that the

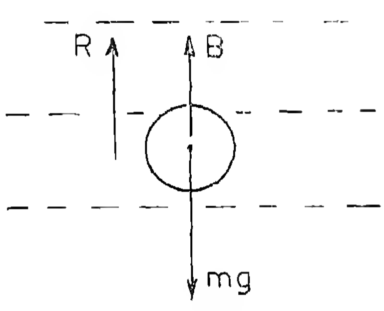


Fig. 12.14 Motion of a sphere through a fluid

resisting force will depend upon (i) the size of the body (ii) the relative speed of the body with respect to the liquid and the viscosity. Thus R, the resisting force on the body is given by

$$R = \text{constant } r^a v^b \eta^c$$

where a, b and c are respectively the powers of radius r, velocity v and viscosity η. From dimensional analysis we see that the dimensions on left hand side must be equal to the dimensions on the right hand side of the equation. That is,

$$MLT^{-2} = (L)^a (LT^{-1})^b (ML^{-1}T^{-1})^c$$
$$= (M)^c (L)^{a+b-c} (T)^{-b-c}$$

Hence $a=b=c=1$. We get

$$R = \text{constant } \eta r v$$

The constant is actually equal to 6π and thus we can write the resisting force as

$$R = 6\pi \eta r v \qquad \ldots (12.8)$$

This equation is called Stoke's law. We shall consider it briefly in relation to a sphere of density ρ falling through a viscous fluid of density ρ_o. The forces acting on the sphere are:

(i) Weight of the sphere (acting downward)

$$mg = \frac{4\pi}{3}\rho \; r^3 g$$

(ii) Force of buoyancy (acting vertically upward)

$$B = \frac{4\pi}{3} r^3 g \rho_o$$

(iii) Resisting force of liquid

$$R = 6\pi\eta rv.$$

The resultant acceleration 'a' with which the sphere is falling downward is given by

$$ma = mg - B - R$$
$$a = g - \frac{(B+R)}{m}$$

If the sphere is released from rest (v=0) the viscous force R at the start is zero. The initial acceleration is therefore

$$a = g\,\frac{\rho - \rho_o}{\rho}$$

As a result of this acceleration, the sphere acquires a downward velocity and therefore experiences a retarding force given by Stoke's law. With increase in velocity the viscous resistance increases. An ultimate constant velocity will be attained by the sphere when the downward acceleration is reduced to zero, and the sphere moves with a constant velocity called its terminal velocity.

Putting a=0 in equation, we get

$$\frac{4}{3}\pi r^3 \rho_o g = \frac{4\pi}{3} r^3 \rho_o g + 6\pi\eta rv$$

$$\text{or} \quad V = \frac{2}{9} \frac{r^2 g}{\eta} (\rho - \rho_o) \qquad \ldots (12.9)$$

Equation holds provided the velocity is not so great that turbulence sets in In presence of turbulence the resistance offered to the motion of sphere is much greater than that given by Stoke's law.

EXAMPLE 12.5

Find the terminal velocity of a steel ball 2 mm in diameter, falling through glycerine. Sp. gr. of steel=8

Sp. gr. of glycerine=1.3
viscosity of glycerine=8.3 poise.
The terminal velocity is given by

$$V = \frac{2}{9} \times \frac{(.1)^2 \times 980}{8.3} (8\ 3 - 1.3)$$

$$= 1.8 \text{ cm sec}^{-1}$$

This method is one of the methods used for measuring viscosity.

12.9 Bernoulli's Equation

Every liquid has a viscosity. The equation of motion of a viscous fluid is very complicated. But it is seen that many physical phenomena are explained qualitatively and to a good approximation even when we neglect the effect of viscosity. Let us now consider an incompressible, non-viscous liquid having a steady flow through a pipe. Let the pipe have a uniform cross-section A_1 at PQ at a height h_1 and a uniform cross-section A_2 at RS at a height h_2 (Fig. 12.15). Let at A_1 the liquid pressure be p_1 and the liquid velocity be V_1. In a small time interval $\triangle t$ the layer at PQ is pushed to P'Q' under the influence of a force p_1A_1. Thus the displacement of PQ is $l_1 = V_1 \triangle t$.

Hence, the work done on the system is

$$W_1 = p_1 A_1 l_1$$
$$= p_1 A_1 V_1 \triangle t$$

The layer at RS advances to R′ S′ by a distance $l_2 = V_2 \triangle t$. If the liquid pressure and velocity

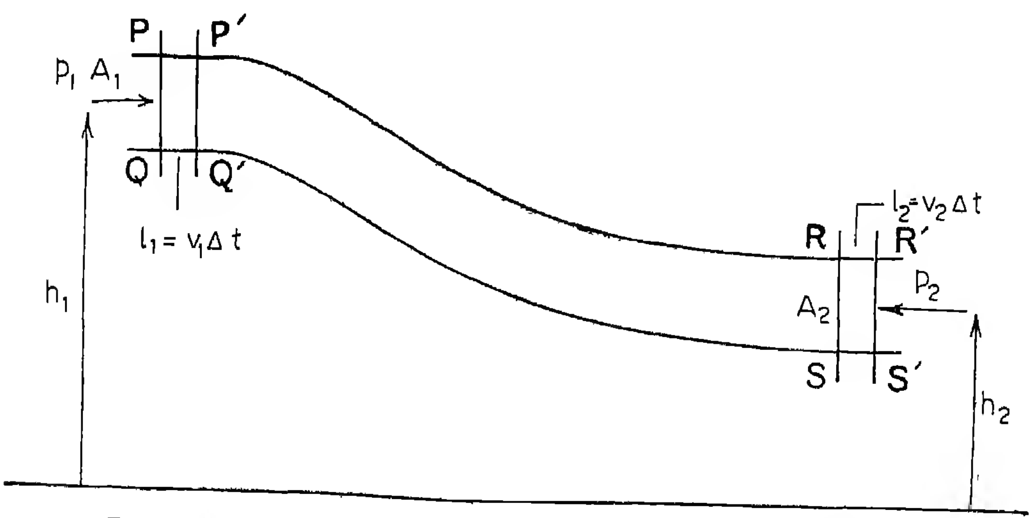


Fig. 12.15 Flow of liquid through a pipe with its ends at different heights

are respectively p_2 and v_2 at A_2, the work done by the system is :

$$W_2 = p_2 A_2 l_2$$
$$= p_2 A_2 v_2 \triangle t.$$

The net work done by the liquid pressure is

$$W = W_1 - W_2$$
$$= p_1 A_1 l_1 - P_2 A_2 l_2$$
$$= (p_1 - p_2) V$$

Hence V $(= A_1 l_1 = A_2 l_2)$, is the volume of the liquid that flows through the pipe in time $\triangle$ t. From the above we see that

$$V = A_1 v_1 \triangle t = A_2 v_2 \triangle t$$
$$\text{i.e. } A_1 v_1 = A_2 v_2 \quad \ldots (12.10)$$

Equation 12.10 is called the equation of continuity. It shows that the speed of liquid flow is inversely proportional to the area of cross-section of the pipe.

During the time $\triangle$ t, the height of the mass ρV changes from h_1 to h_2. The work done on the liquid by gravitational field is

$$W_g = -(h_2 - h_1) \rho V g$$
$$= \rho g V (h_1 - h_2)$$

We know that when work is done on an element of mass ρV, its kinetic energy changes. Hence

change in K.E. $= W_p + W_g$
= Total work done on the system

$$\text{or } \frac{1}{2}\rho V \, (v_2^2 - v_1^2) = (P_1 - P_2)V + \rho g V (h_1 - h_2)$$

$$\text{or } \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_2$$

$$\text{or } \frac{p}{\rho g} + \frac{v^2}{2g} + h = \text{constant} \quad \ldots (12.11)$$

Equation 12.11 is Bernoulli's equation. Every term in this equation has dimensions of length and is called a head. The $p/\rho g$ is called pressure head, $v^2/2g$ the velocity head and h the gravitational head. We may state, 'where the velocity is high the pressure is low, and where the velocity of a fluid is low the pressure is high.' We can discuss some illustrative examples of Bernoulli's principle.

(i) During certain wind storm or cyclone the roof of some houses are blown off without damaging the other parts of house (Fig. 12.16). This is not a freakish accident as

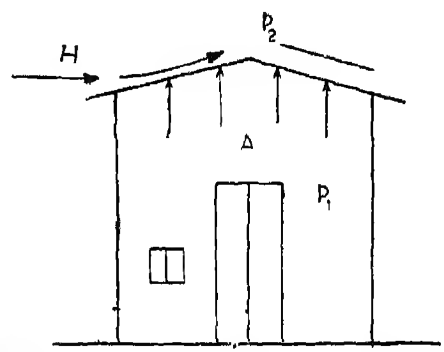


Fig. 12.16 Atmospheric pressure p_1 and pressure p_2 on the top of the roof causes the roof to be lifted, H-wind with high velocity

one might think, for there is a simple explanation. A high wind blowing over the roof, creates a low pressure p_2 on top. Under the roof the pressure is p_1 (=atmospheric pressure) which is larger than p_2. The difference of pressure $(p_1 - p_2)$ causes an upward thrust and the roof is lifted up. Once the roof is lifted it is blown off with the wind.

(ii) In a common form of atomizer, squeezing the bulb spray out oil or scent from the nozzle (Fig. 12.17). The rise of liquid is not due to any vacuum created in the central tube. When the bulb is squeezed the air blows through the central tube with high speed causing a low pressure p_2 inside. The atmosphere pressure p_1 on the liquid

surface of the container pushes the liquid up the steam which then is blown out with the air steam through the nozzle.

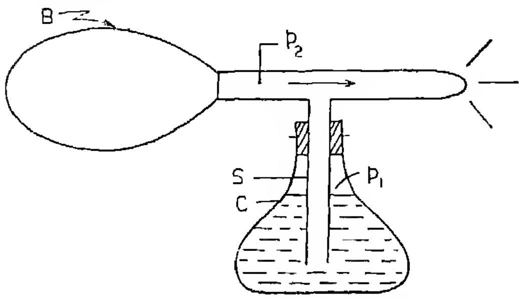


Fig. 12.17 Atomizer; B-bulb, p_1-atmospheric pressure; C-container; S-stem; P_1-pressure inside

(*iii*) *Lift on an aircraft wing*: Figure 12.18 shows the cross-section of the wing of

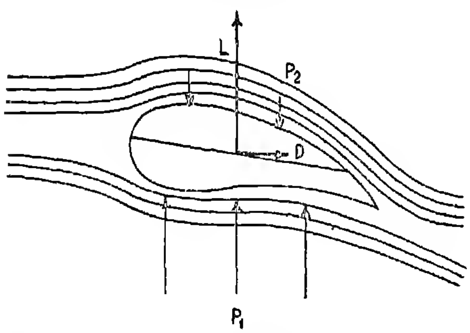


Fig. 12.18 Profile of a wing and lines of air flow

an aircraft. When the aircraft moves, the air flows both from under and above the wing. Aeroplane wings are so designed that the total distance travelled by air flowing over the wing is longer than that of the air flowing under it. Thus, the velocity of air flow above the wing must be higher than the velocity of air flow under the wing. From Bernoulli's theorem we see that the pressure p_2, above the wing is lower than the corresponding pressure p_1, under the wing. The unbalanced pressure causes a force F to act on it This force can be resolved into two components. The vertical component is responsible for the lift L and the horizontal component provides the drag D. The force acting on it depends on angle of attack If angle of attack is too great, the streamline flow above and behind the wing breaks down and a complicated system of whirls and eddies known as turbulence is set up. Bernoulli's equation no longer applies. The pressure above the wing rises and the lift on the wing decreases. The plane stalls.

There are various other examples which can be observed by the reader in his daily life where Bernoulli's principle is used.

(*iv*) *Curved path of a spinning ball*: When a ball is thrown or hit with a horizontal velocity v and given a spin the path of the projectile of the ball curves more than in a usual spin free projectile. Let a cricket ball moving to the right be given a spin as shown in Figure 12.19 (a). This situation

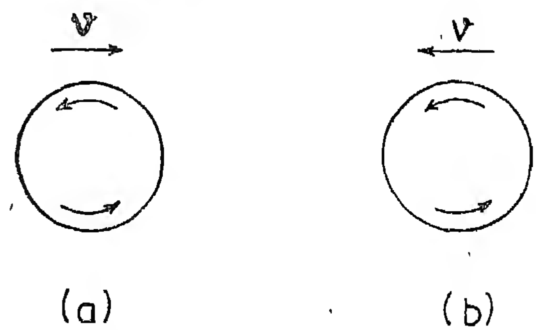


Fig. 12.19 Motion of a spinning cricket ball

is effectively the same as the ball spinning with air wind moving in opposite direction of v (Fig. 12 19 (b). Thus just above the ball the wind has a velocity greater than v as the velocity componant due to spin adds up to v and just below the opposite is the case. A pressure difference of air created

at the top and bottom of the spinning ball, produces an additional unbalanced force. This force divides the spinning ball from the path of a free projectile. If the spin is in opposite sense pressure difference above and below the ball is reversed and the ball curves more downwards.

Exercises

12.1 A liquid drop of diameter D breaks up into 27 tiny drops Find the resulting change in energy. ($2\pi D^2\sigma$)

12.2 What would be the gauge pressure inside an air bubble of 0 20 mm radius situated just below the surface of water ? Surface tension of water is 0 07 Nm^{-1}. ($14\times 10^{-2}\ Nm^{-2}$)

12.3 A U tube is such that the diameter of one limb is 0.4 mm and (Hint · unlike soap bubble there is only one surface here) that of the other is 0.8 mm. Find the difference in the level of water in the limbs. ($\sigma 0.07 Nm^{-2}$)

12.4 Why is it that a needle may float on clear water but will sink when some detergent is added to water ?

12.5 At what speed will the velocity head of a stream of water be equal to 40 cm of Hg ? (2 8 m/s)

12.6 If a small ping pong ball is placed in a vertical jet of air or water, it will rise to a given height above the nozzle and stay at that level. Explain.

12.7 A pilot-tube is mounted on an aeroplane wing to measure the speed of the plane (Fig. 12.20). The tube contains alcohol and shows a level difference of 40 cm. What is the speed of the plane relative to air (sp.gr. or alcohol= 0.8 and density of air 1 kgm^{-3}) ?

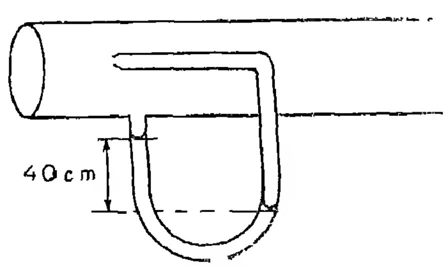


Fig 12.20

12.8 A glass tube of 1 mm bore is dipped vertically into a container of mercury, with its lower and 2 cm below the mercury surface. What must be the

gauge pressure of air in the tube to blow a hemispherical bubble at its lower end.

12.9 Water stands at a depth H in a tank whose side walls are vertical (Fig. 12.21). A hole is made on one of the walls at a depth h, below the water surface.

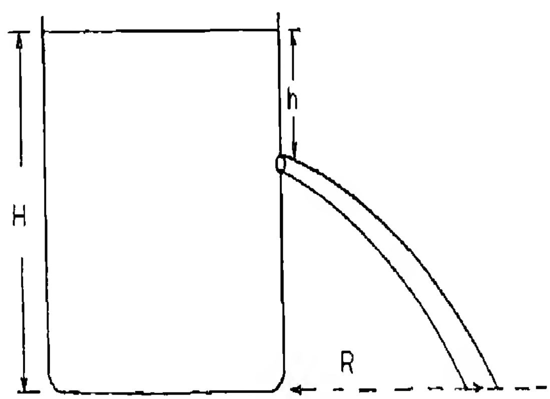


Fig. 12 21

(a) At what distance R from the foot of the wall does the emerging stream of water strike the floor ?

(b) For what value of h this range is maximum ?

12.10 With what terminal velocity will an air bubble 0 8 mm in diameter rise in a liquid of viscosity $0.15 Nm^{-2}s$ and specific gravity 0.9 ? What is the terminal velocity of the same bubble in water ?

12.11 A sphere is dropped under gravity through a fluid of viscosity η. Taking the average acceleration as half of the initial acceleration, show that the time to attain the terminal velocity is independent of the fluid density.

UNIT 13

Electricity

13.1 Electric Current

The term current implies some sort of motion. A motion of electric charge constitutes an electric current. This motion may be that of the charge carriers in a medium, like electrons in a metal and ions in a liquid or a gas, or it may be the motion of electrons in a vacuum tube.

Quantitatively, electric current is defined as the rate of flow of charge, given by

$$I = q/t \qquad \text{... (13.1)}$$

where I is the current and q is the charge that has passed through a given area in time t.

If the rate of flow of charge does not change with time the current is said to be steady. In many situations, however, the current may be varying with time. Thus in Fig. 13.1, curve (a) represents a steady current while curves (b) and (c) represent varying currents.

By convention, the direction of motion of positive charge is taken as the direction of current. A negative charge moving in one direction is equivalent to an equal positive charge moving in the opposite direction (Fig. 13.2).

In S.I. units the current is measured in amperes, 1 ampere = 1 coulomb/second. It is interesting to note that in practice the electric currents are not measured directly by measuring charge and time but by the effects produced by them, for instance, the magnetic effects.

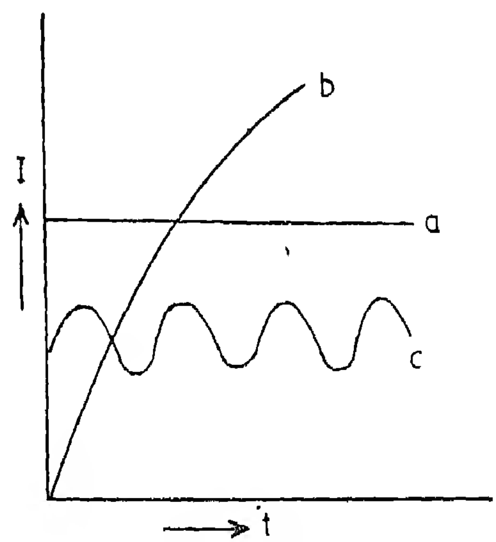


Fig. 13.1 Examples of steady and varying currents

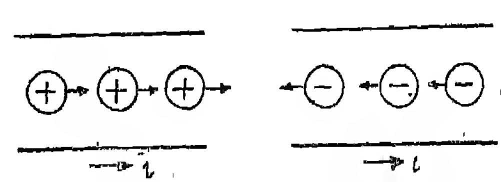


Fig. 13.2 Flow of positive charge is equivalent to flow of negative charge in the opposite directions.

EXAMPLE 13.1

One billion electrons pass from a point A towards another point B in 10^{-3} seconds. What is the current in ampere ? What is its direction ?

Solution

Charge on 1 electron $= 1.6 \times 10^{-19}$C

$q = 1.6 \times 10^{-19} \times 10^{9}$

$= 1.6 \times 10^{-10}$C

$t = 10^{-3}$ s.

$I = q/t = 1.6 \times 10^{-10}/10^{-3}$

$= 1.6 \times 10^{-7}\,\text{Cs}^{-1}$

$= 1.6 \times 10^{-7}$ A.

Since the electrons are negatively charged, the direction of current flow is from B to A.

EXAMPLE 13.2

The electron in the hydrogen atom circles around the proton with a speed of 2.18×10^{6} m/s in an orbit of radius 5.3×10^{-11} m. Find the equivalent current.

Solution

Radius of orbit, $r = 5.3 \times 10^{-11}$ m.

Circumference of the orbit$=2\pi r$

$=2\pi \times 5.3 \times 10^{-11}$ m

Speed of electron, $v = 2.18 \times 10^{6}\ \text{ms}^{-1}$

In one second the electron will go around the proton n times, where

$$n = \frac{v}{2\pi r} = \frac{2.18 \times 10^{6}}{2 \times 3.14 \times 5.3 \times 10^{-11}}$$

The electric current is defined as the quantity of charge that flows past a fixed point in one second. Although it is the same electron going round and round the proton, the quantity of charge that passes across a fixed point on the orbit in one second is given by ne, where e is the electronic charge.

Hence the equivalent current

$$I = ne = \frac{2.18 \times 10^{6}}{2 \times 3.14 \times 5.3 \times 10^{-11}} \times 1.6 \times 10^{-19}\,\text{Cs}^{-1}$$

$$= 1.05 \times 10^{-3}\ \text{A}.$$

Although we are more familiar with currents in a wire, currents can also flow through liquids and gases. The only requirement is the availability of charge carriers and a difference of potential. A mobile positive charge carrier would then move from a region of high potential to a region of low potential under the influence of the electric field, a negative charge carrier in the opposite direction. In solid conductors, the charge carriers are mostly electrons, in liquids and gases it is the positive and negative ions (in gases, electrons also) that contribute to the current flow.

Consider a pair of charged plates (Fig. 13.3),

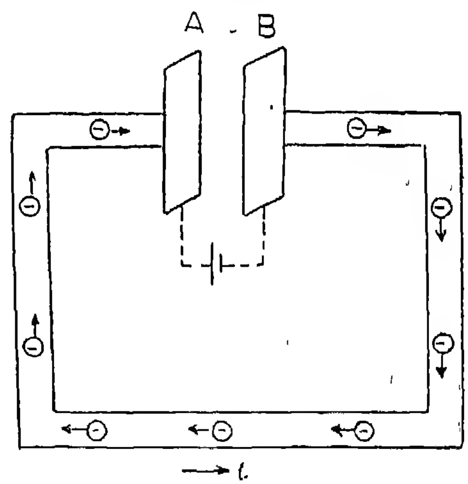


Fig 13.3 Steady current requires a source of constant potential difference

plate A being at the higher potential. If we connect the plates by a wire, electrons will travel from plate B to plate A and an electric current would be said to flow from A to B, through the wire. Now, the arrival of electrons at the plate A will tend to equalise the potential between the plates and, unless the potential difference is maintained by external means, the current would soon cease to flow. Thus, for steady currents to flow between two points

the potential difference between them must be maintained by a battery or any other device.

Current I through a conductor depends upon the potential difference V applied across it. By changing V, one can study the variation of I with V. Fig. 13 4 show such plots for

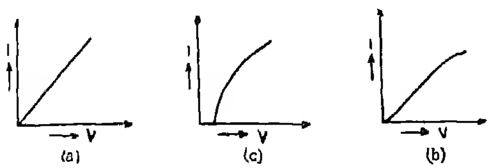


Fig. 13.4 V-I plots for (a) metallic conductor (b) vaccum tube (c) electrolyte

(a) a metallic conductor, (b) a vacuum tube, and (c) an electrolyte. The ratio

$$\frac{V}{I} = R \qquad .. (13.2)$$

is known as the resistance of the conductor.

We see that for the metallic conductor, the V-I curve is a straight line i.e. the resistence R of the conductor is independent of V. In this case we say that the conductor obeys Ohm's law. All such conductors which show this kind of linear behaviour are known as ohmic resistors. Others, for which the V-I curve is not a straight line are called non-ohmic resistors. Both kinds are used in electric circuits for various purposes.

13.2 Current Flow in a Metallic Conductor : Draft Velocity

Let us examine in some detail the mechanism of current flow in a metallic conductor.

In a metal in solid state, atoms occupy places in a fixed arrangement. In this state some electrons are free, leaving the atoms as positively charged ions. If we assume that there is one free electron per atom, there will be of the order of 10^{28} free electrons per cubic metre. Since they possess thermal energy, they are in a state of continual motion, much like the molecules of a gas. They are colliding with atoms, gaining and losing kinetic energy At room temperature they move with velocities of the order of 10^5 ms^{-1}, but these velocities are randomly distributed in all directions. Therefore, the average velocity of electrons, given by

$$u = \frac{u_1+u_2+u_3+\quad+u_N}{\phi N}$$
$$=0$$

where u_1, u_2 etc. are velocities of individual electrons and N is the total number of electrons

In other words, there is no net flow of charge in any direction. It may be noted that average speed is not zero.

When a potential difference is applied across the conductor, an electric field E is established inside it. Due to the electric field the electrons experience coulomb forces in a direction opposite to the field and they undergo an acceleration ($a = -\frac{eE}{m}$). The forces are experienced by positive ions also but they are unable to move as they are heavy and tightly bound in the metal. Between two successive collisions an electron thus acquires, in addition to its thermal velocity, a velocity component in a direction opposite to E. This gain in velocity due to the field E, however, is very small and is lost in the next collision with an atom.

If we looked at any arbitrary instant, we would find an electron moving with a velocity u_1+at_1, where u_1 is its velocity due to thermal motion and at_1 is the velocity acquired under the influence of the external field E, t_1 being the time that has elapsed since the last

collision The average velocity of all electrons will be given by

$$V = \frac{(u_1+at_1)+(u_2+at_2+.)......+(u_N+at_N)}{N}$$

$$= \frac{u_1+u_2+ \quad ...u_N}{N} + \frac{a\,(t_1+t_2+......t_N)}{N}$$

$$= 0+at, \text{ where } t = \frac{t_1+t_2+. \quad ... \,. t_N}{N}$$

t represents the average time elapsed since each electron suffered its last collision, it is called the relaxation time and is $= 10^{-14}$ s. It is a characteristic of the given conductors. Denoting v by V_d

$$V_d = -e\,E\,t/m \qquad ...(13.3)$$

V_d is called the drift velocity of electrons.

We may say that before the application of the field, the average velocity of the electrons is zero and there is no net transport of charge in any direction. After the field is applied, electrons acquire a constant average drift velocity in a direction opposite to E and there is a net transport of negative charge in that direction which is equivalent to an electric current flowing in the direction of E. A typical value of V_d is 1 mm/s, which is very small compared to the random velocity (10^6 m/s) But, since the number of electrons is very large, even this small drift of electrons adds up to a sizeable current.

One sometimes tends to confuse between the drift speed of electrons and the speed of propagation of electrical effects. It may look surprising that although the drift velocity of electrons is so low, an electric bulb turns on almost immediately when it is switched on The fact is that the propagation of an electric impulse takes place with a speed which is of the order of speed of light ($=3\times10^8$m). It is somewhat like applying a pressure to one end of a long water filled tube. As soon as the pressure is applied a pressure wave is transmitted rapidly along the tube and when it reaches the other end the flow of water starts The water inside the tube also starts moving forward but this speed is much slower than the speed of pressure wave Similarly in a circuit, free electrons are present everywhere When a potential difference is applied to the circuit, an electric field gets established throughout the circuit almost with the speed of light. As soon as the electric field is established the electrons begin to drift under its influence and a current flows in the circuit.

Resistance

Consider a conductor of length l and of uniform cross section A, across which is applied a potential difference V, Fig. 13.5. The electrons start drifting towards left with a

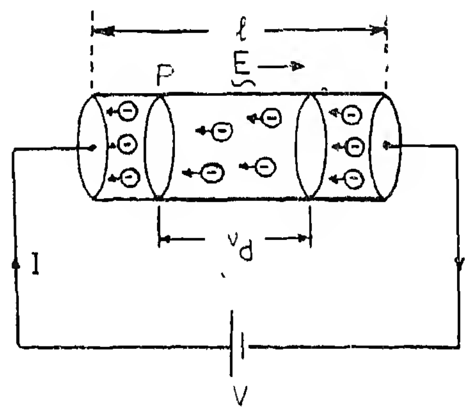


Fig. 13.5 Flow of current through a conductor

velocity V_d. All those electrons which lie within a distance V_d on the right of a cross section at P, would have passed on to the other side in one second. The net transport of charge through the section at P in 1 second or the current I is, therefore, given by

$$I = enAV_d \qquad ... (13\ 4)$$

where n is the number of free electrons per unit volume in the conductor It is easily seen,

using equation (13.3), that

$$I = enA \cdot \frac{eEt}{m}$$

$$= \frac{(e^2nAt)}{ml} V, \text{ since } E = V/l.$$

Hence, the resistence,

$$R = \frac{V}{I} = \frac{ml}{e^2nAt} \qquad (\dots 13.5)$$

R is a constant for the given conductors. We can also write

$$R = \rho\, l/A, \qquad \dots (13.6)$$

Where $\rho = m/e^2nt$ .. (13.7)

is called the resistivity of material of the conductor. The resistivity is a constant of the material of the conductor It can be varified that units of P are ohm-metre.

EXAMPLE 13.3

What is the drift velocity of electrons in a copper conductor having a cross sectional area of $5 \times 10^{-6} m^2$ if the current is 10A. Assume there are 8.0×10^{28} electrons/m^3.

Solution

$$I = en\, AV_d$$

Here, $e = 1.6 \times 10^{-19} C$, $n = 8.0 \times 10^{28}/m^3$, $A = 5 \times 10^{-6} m^2$, $I = 10$ A

$$V_d = \frac{I}{enA}$$

$$= \frac{10}{1.6 \times 10^{-19} \times 8.0 \times 10^{28} \times 5 \times 10^{-6}}$$

$$= 1.56 \times 10^{-4} \text{ m/s.}$$

13.3 Heating Effects of Current : Joule's Law

It is a matter of common observation that when a current flows in a conductor heat is developed. This can be understood on the basis of the microscopic picture presented above. We have seen that the motion of electrons in a conductor under the influence of the field is hampered by their frequent collisions with the atoms; this being the basic cause of resistance. In between the collisions, the electrons moving through the field, acquire kinetic energy in addition to their own kinetic energy of thermal motion. In the collisions this energy is shared with the atoms of the metal with a consequent increase in the average kinetic energy of the atoms. The conductor is thus heated by the flow of electric current through it.

Consider the portion AB of a circuit (Fig. 13.6) through which a current I is flowing in the

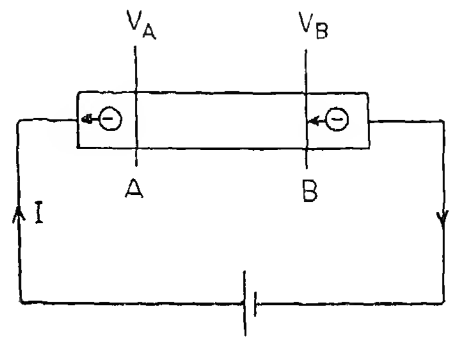


Fig 13.6 For calculation of power dissipation in a conductor

direction A to B Electrons enter at B at potential V_B and leave at A at potential V_A. Since electrons for negatively charged and $V_A > V_B$ the electrons entering at B possess more potential energy than those leaving at A, while their kinetic energies are the same. The electrons thus lose potential energy while passing from B to A. It is this difference in potential energy which is transferred to the conductor as heat energy. An expression for the same can be drived as follows:

The potential energy of an electron at B $= (-e)\, V_B$.

The potential energy of an electron at A $= (-e)\, V_A$.

Number of electrons which enter the portion AB at B and leave it at A, in tim $dt = Idt/e$

The loss in potential energy of electrons in time dt is

$$dw = \frac{Idt}{e} [(-e)V_B - (-e)V_A]$$
$$= Idt\ (V_A - V_B)$$
$$= Idt\ V_{AB} \text{ where } V_{AB} = V_A - V_B$$

This energy appears as heat. Therefore, the rate at which the electric energy is converted into heat, that is, power P is given by

$$P = \frac{dw}{dt} \quad V_{AB}\ I \qquad .. (13.8)$$

Since V_{AB}=IR, where R is the resistence of the portion AB of the conductor, we have

$$P - V_{AB}I = (IR)\ I = I^2R \qquad .. (13.9)$$

This is Joule's law of heating. If V_{AB} is in volts and I in amperes, then P is in watts.

$$1 \text{ amp} \times 1 \text{ volt} = \frac{1\text{coulomb}}{\text{second}} \times \frac{1 \text{ joule}}{\text{coulomb}}$$
$$= \frac{1 \text{ joule}}{\text{second}} = 1 \text{ watt.}$$

Heat developed in time t is given by

$$Q = Pt = I^2 Rt \text{ joules} = \frac{I^2Rt}{4.2} \text{calories} \quad . (13\ 10)$$

Since 1 calorie = 4.2 joules.

The kilowatt hour : If P is power in watts and t is time in seconds, the energy consumed is

W=Pt joules

For consumption of electric energy, joule is considered to be too small a unit for practical purposes such as calculating monthly electricity bill The practical unit for electric energy is *kilowatt-hour*, defined by

W=Pt

where, P is in kilowatts, t is in hours and W in kilowatt hours. Since 1kw=1000w, and 1 hour =3600 seconds, we have,

$$1\text{kwhr} = 1000 \times 3600 = 3.6 \times 10^6 \text{ joules.}$$

EXAMPLE 13.4

An electric heater is rated at 800 watts.

(i) If the voltage is 200 volts what is the value of current ?

(ii) How much time would be required to heat 1 litre of water from 20°C to 100°C ?

Solution

(i) I=P/V = 800/200=4.0A

(ii) Heat required=mass of water×specific heat of water×rise in temperature
=(1000gm) (1 cal/gm°C) (80°C)
=80,000 cal.=80,000×4.2 joules.

$$\text{Therefore, } t = \frac{Q}{P} = \frac{80{,}000 \times 4.2}{800} = 420\text{s.}$$

EXAMPLE 13.5

What voltage drop will be there across a 1kw electric heater element whose resistance when hot is 40 ohms ?

Solution

$$P = I^2R = (V^2/R^2)R = V^2/R$$
$$\therefore V = (P \times R)^{\frac{1}{2}} = (1000 \times 40)^{\frac{1}{2}} = 200 \text{ volts.}$$

13.4 Variation of Resistance with Temperature

It has been stated before that the atoms in the metal occupy fixed positions. They are, however, not stationary but vibrate to and fro about their fixed positions. When a metal gets heated there is an increase in the average kinetic energy of the atoms and they vibrate more vigorously. This leads to more collisions of electrons with atoms and a consequent decrease in the value of t. Also at highert emperature the thermal speed of electrons is more and this further reduces the time between collisions. A lower value of t, therefore, results in an increase in the resistivity of the conductor equation (13 7.)

Experimentally it is found that to a first approximation

$$R_t = R_0 (1 + \alpha t) \quad \ldots (13.11)$$

where R_t and R_0 are resistances at temperatures t°C and 0°C respectively, and α is known as the temperature coefficient of resistance For metals α is a positive coefficient and for copper its value is 0 0040 per degree. For some alloys, like constantan and manganin the value of α is very small (0.00001) and these are used in making standard resistances Carbon and other semiconductors display a negative coefficient i.e. their resistance decreases with temperature α is negative for electrolytes also. It is beyond the scope of this book to go for an explanation of negative α.

13.5 Thermo-electric Effects

Fig. 13.7 shows two pieces of wire, one copper and the other iron, joined together at

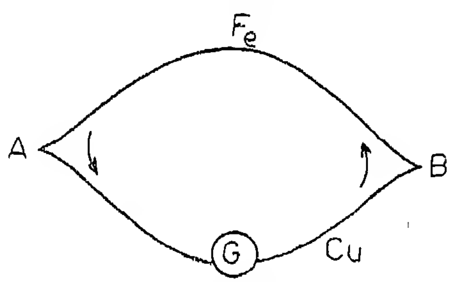


Fig. 13.7 A thermo couple · A is hot Junction and B is cold junction

their ends A and B. A low resistance galvanometer G is included in the circuit. When one junction is heated and the other kept at a constant temperature, a small electric current flows around in the circuit in the direction of arrows. This is known as seeback effect after its discoverer. The effect is exhibited by any two dissimilar metals joined in this manner. Seeback investigated the effect for a number of metals and arranged them in a series such that when any two of them form a circuit the current flows at the hot junction from the metal occurring earlier in the series to the one occurring later. Some of the substance in the series are Bi, Ni, Pt, Cu, Rh, Ir, Fe Sb.

Current produced in this way is known as thermoelectric current and the pair of metals forming the circuit is called a thermocouple. Evidently the thermoelectric current flows because an e.m f. (electromotive force) has been set up in the circuit purely due to thermal effect This e m.f. is known as thermo e.m f and can be measured by using an instrument called a potentimeter. It is a normal practice to measure this e.m f. by keeping one junction at a fixed temperature, very often at O°C, and varying the temperature of other junction. A typical curve is shown in Fig. 13.8.

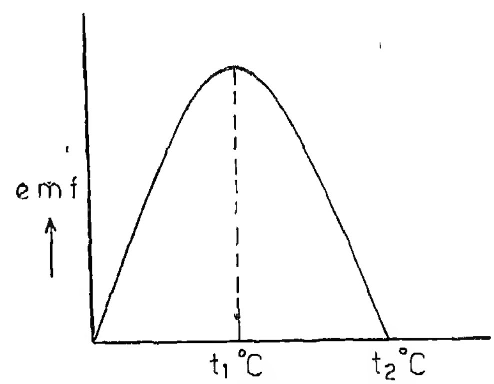


Fig 13.8 Variation of thermo e.m.f. with temperature

The maximum e.m f. is obtained at t_1°C which is called the neutral temperature of thermocouple. At t_2°C the thermo e.m.f. falls to zero value and reverses its direction. This temperature called the inversion temperature, is not unique but depends upon the temperature of cold junction. It is always as much above the neutral temperature as the cold

junction is below it. For Cu-Fe couple, the neutral temperature is about 300°C.

13.6 Temperature Measurement by Thermocouples

Thermocouples are widely used for temperature measurement. If the temperature of cold junction is fixed, the thermal e.m f. of a thermocouple is a function of the temperature of hot junction alone. For a considerable range of temperature this e.m f. is a linear function of temperature. This fact can be used for measuring the temperature at the site of the hot junction if the couple is previously calibrated. Frequently the galvanometer in the circuit is calibrated to read directly the temperature. The choice of thermocouple wires depends upon the range of temperature required. The copper-constantan couple, consisting of copper and an alloy called constantan, is used to measure temperature ranging from −190°C to 300°C. Thermocouples of platinum and a platinum-rhodium alloy are used for measuring temperatures upto 1600°C.

As a temperature measuring device, the thermocouple has many merits. It is accurate. The copper-constantan couple, for example, gives an e.m.f. of 40 microvolts per degree temperature difference between its junctions. This e.m.f. can be measured with an accuracy of about 1 microvolt. The temperature differences, therefore, can be measured with an accuracy of about 1/40 degree Another great advantage of a thermocouple is the smallness of its test junction. This makes it particularly suitable for measuring temperature in very small regions and cavities. Because of its small mass the test junction comes quickly into thermal equilibrium with the surroundings. Use is made of this fact in measuring variations in the temperature in various parts of the body of animals and insects.

Thermopile

The e.m.f. generated by a thermocouple is small, of the order of few millivolts. But, if a number of thermocouples are connected in series the e.m.f.s are added up. Such a combination is called a thermopile and is used to measure the intensity of heat radiation.

One set of junctions (A) is exposed to the radiation, the other set (B) is protected by an insulating lid (Fig. 13.9). The exposed ends

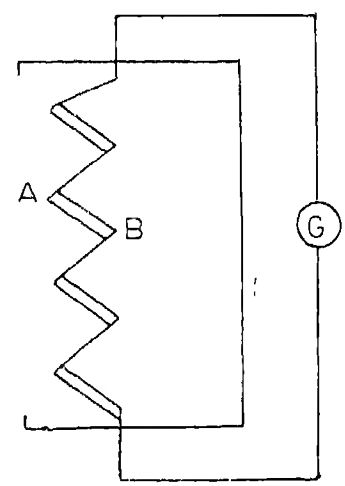


Fig. 13.9 Thermopile

are blackened to increase the absorption of radiation falling on them. The thermocouples are made of antimony and bismuth strips. The galvanometer deflection is proportional to the intensity of falling radiation.

13.7 Magnetic Effects of Electric Current

Before we discuss the magnetic effects of current as such, we discuss the nature and meaning of electric and magnetic fields.

The forces and the fields

When a body exerts a force on another

body and vice versa, the two bodies are said to interest with each other. Thus, the earth and the moon interact to exert mutal forces on each other. These forces are given by Newton's law of gravitation. Another kind of interaction called electric interaction occurs between two charged bodies, the force that one charged body exerts upon the other is given by Coulomb's law. We are also aware of the force that one magnet exerts on another. This is called magnetic interaction. However, this is not a new kind of interaction. As we shall presently see, the magnetic forces can be explained in terms of electrical forces between moving charges. The common interaction which describes both electric and magnetic forces is called, aptly the electromagnetic interaction.

In the example cited above, there is one common feature—these are examples of 'action at a distance.' The force is transmitted from one objected to another without any direct contact. In contrast most of the forces that we come across in daily life seem to involve direct physical contact e.g. pushing a body, pulling a string etc. How do the gravitational, the magnetic and the electric forces transmit themselves from one object to another ?

The interaction between two bodies through intervening space can be understood by introducing the concept of 'field'. We illustrate it by taking the examples of force between two charges.

The experimental fact that an electric charge experiences a force due to another charge located some distance away can be understood as follows. We say that one of the charges sets up an electric field in the surrounding space, this field acts on the other charge and produces the observed force. Since the force is mutual, we may regard any one of the charges to be a source of the field which acts upon the other. The electric field thus acts an intermediary for interaction between two charged bodies.

Similarly, the two masses are supposed to interact via a gravitational field and the two magnets through a magnetic field. The origin of magnetic field can, however, be traced back to the moving charges as discussed later.

The electric field

Consider a charge q situated at some point in space. It creates an electric field around it. We call q, the source charge. If another charge q_o, which we call the test charge, is placed at some point in this field (Fig. 13. 10(a), it experiences a force given by

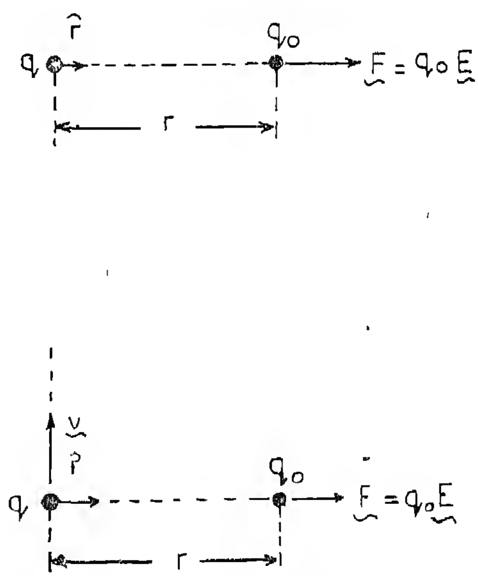


Fig. 13.10 (a) Force on a test charge due to a stationary charge
(b) Force on a test charge due to a moving charge (V<<C)

$$F = q_o\ E \qquad ...13.12)$$

where **E** is the electric field at that point.

To relate the value of **E** to the source charge

we use Coulomb's law which gives the force **F** between q and q_o as

$$\mathbf{F}= \frac{q\, q_o}{4\pi\epsilon_o\, r^2}\mathbf{r} \quad \text{..(13.13)}$$

where **r** is the unit vector from q to q_o and $\mathbf{E}_o$ ($=8.85\times10^{-12}C^2$ N—m^2) is the permittivity of free space. The electric field E, due to q at the site of q_o is, therefore, given by

$$\mathbf{E}= \frac{q}{4\pi\epsilon_o\, r^2}\mathbf{r} \quad \text{...(13.14)}$$

In the above discussion the source of the field was a stationary charge. It is found that a moving charge is also a source of electric field. If the charge q is moving (Fig 13.10 b) then the force on a stationary charge q_o is still given by $\mathbf{F}=q_o\mathbf{E}$, where **E** is the electric field due to q at the site of q_o at the instant of observation For small velocities* of q, the field is the same as due to a stationary q, given by equation (13.14)

The electric field obeys superposition principle. The field produced at a certain point in space when there are number of sources is the vector sum of the fields that each source would individually produce if it alone were present.

The force equation, $\mathbf{F}=q_o\,\mathbf{E}$, enables us to define the electric field at a point without reference to its sources. If a test charge placed at rest at a point in space experiences a force, we say an electric field exists at that point. The strength of the field is the force per unit charge on the test charge and its direction is the same as that of the force acting on a (positive) test charge, In symbols,

$$\mathbf{E}=\frac{\mathbf{F}}{q_o} \quad \text{...(13.15)}$$

An important property of the electric field is this: The force that it exerts on a test charge is independent of the velocity of test charge. Thus, if there is an electric field **E** at a point, the force exerted on a test charge q_o at that point will be $q_o\,\mathbf{E}$, irrespective of the fact whether the charge q_o is at rest or in motion.

The Definition of Magnetic Field

Laboratory experiments show that two parallel wires carrying currents in the same direction attract each other (Fig. 13 11(a), if

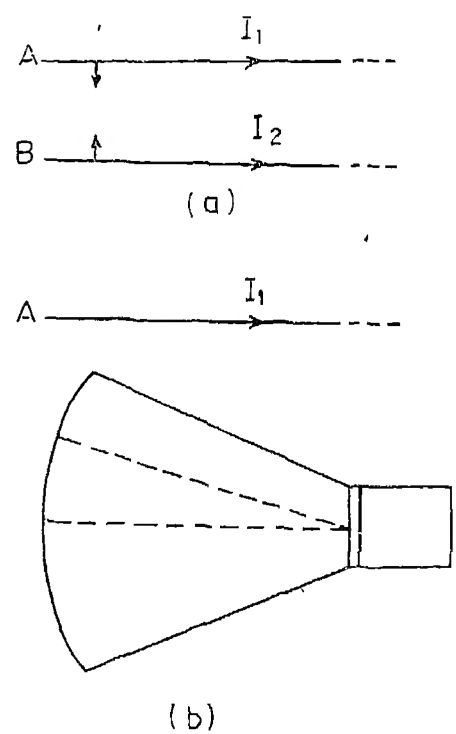


Fig. 13.11 (a) Magnetic interaction between parallel wire carrying currents
(b) Deflection of an electron beam by a magnetic field

* For high velocities (of the order of velocity of light) the field becomes somewhat complicated but we need not to go into these details here,

the currents are in opposite directions, they repel. It is convenient to imagine, that one current produces some kind of field which exerts a force on the other current. Let us investigate the nature of this field.

This field cannot be electrostatic as the wires have no net charge on them because they have equal number of electrons and positive ions. Moreover, if we remove wire B and place a charged conductor there, it experiences no force. This field, therefore, does not interact with stationary charges.

Further, if we switch off i_2, the mutual force of attraction (or repulsion) is also switched off, although current i_1 and its field are still there. It follows, therefore, that the field due to i_1 acts on the wire B only when there is a current through it, that is when there is a flow of charge through wire B.

To test whether the field acts only upon the charges moving in a conductor or it also acts on charges moving freely we replace B with an evacuated discharge tube (Fig. 13 11(b) and find that the electron beam gets deflected (i.e. and force acts on the moving charges). The magnitude of deflection varies with the direction of motion of the charges in the beam. It is found that there is a direction for which the deflection is zero and at right angles to it, the deflection is a maximum. Also, the deflection is found to be proportional to the speed of electrons So, here we have a field which exerts, on a moving charge, a force, which is velocity dependent. We call it the magnetic field and denote it by the symbol B.

If we assume that the direction of magnetic field coincides with the direction of motion of charges when the deflection is zero, we find that, in general, the force on a charge q moving with velocity v can be described by the expersion.

$$\mathbf{F}=q v B \sin\phi \qquad \dots (13.16)$$

where ϕ is the angle between **B** and **v**. Also, it is found that force **F** is in a direction perpendicular to both **B** and **v** This enables us to express F, in vector notation as

$$\mathbf{F}=q(\mathbf{v} \times \mathbf{B}) \qquad .. (13.17)$$

The relationship between **F**, **v** and **B** is shown in Fig. 13 12 It is governed by the

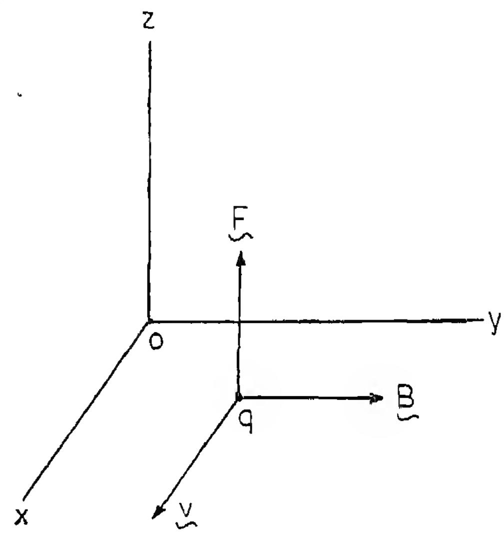


Fig. 13.12 Shows the direction of force on a charge moving in a magnetic field

usual rule for a vector product the direction of **F** will be the one in which a right handed screw will advance by a rotation of **v** towards **B** through the smaller angle between them. Then, if **v** and **B** lie in the x—y plane as shown, **F** will point out in the z direction. [Note that for a beam of electrons q is negative and it follows from equation (13.17) that **F** has the opposite direction to that of (V × **B**).]

Unit of B : In S. I. system, unit of magnetic field B is called Tesla. In the equation,

$$B=\frac{F}{q v \sin\theta} \qquad \dots (13.18)$$

if $F = 1$ newton
$Q = 1$ coulomb
$v = 1$ metre/second
$\phi = 90°$

Then, $B = 1$ newton-second/coulomb-meter= 1 tesla, that is, a charge of 1 coulomb moving with a speed of 1m/s at right angles to a magnetic field of strength 1 tesla will experience a force of 1 newton.

A field of tesla is a very strong magnetic field. The field in the neighbourhood of common permanent magnet is 0.1 tesla The earth's magnetic field on the earth's surface is 5×10^{-5} tesla. Very often the magnetic fields are expressed in term of a smaller unit, called the gauss. 1 gauss$=10^{-4}$ tesla.

In some books **B** is called the magnetic induction.

Source of the magnetic field

We have seen that a current carrying wire produces a magnetic field. If the current is switched off, the magnetic field disappears.

It immediately suggests that the magnetic field is the result of the motion of electrons in the wire One may go further and say, in general, a moving charge is a source of magnetic field.

When there is no current in the wire, the electrons are in thermal motion. Though each electron acts as the source of a magnetic field, these fields cancel out since electrons are moving in a random manner. There is no net magnetic field as there is no net flow of charge in any direction.

13.8 Biot-Savart Law—Magnetic Field due to Some Current Distributions

A magnetic field is produced by a moving charge We are often interested not in the field of an isolated moving charge but that of a current in a conductor. Each of the moving charges within the conductor contributes to the magnetic field outside Experiments show that the direction and magnitude of **B** due to any current depend upon the current direction and the relative position of the observation point. It is more convenient to think that the field is made up of contribution from various segments of the conductor called the current elements rather than from individual charges, in such a case.

The Fig. 13.13 shows a typical current

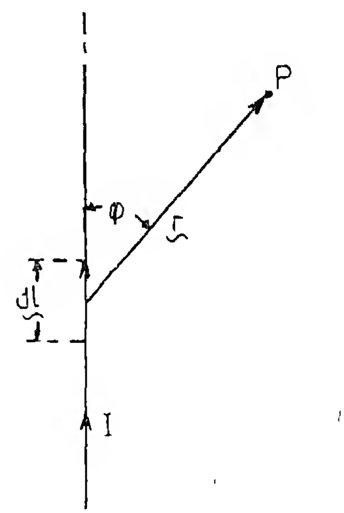


Fig. 13.13 Biot-Savart law

element of length **dl** of a conductor carrying a current I. The magnetic field **dB** at the point P, due to the current **dl** is then given by Biot-Savart Law* :

$$\mathbf{dB} = \frac{\mu_0}{4\pi} I \frac{\mathbf{dl} \times \mathbf{r}}{r^3} \quad \dots (13.19)$$

* This law was arrived at empirically, by Biot and Savart, in 1820, to provide a consistent explanation of numerous experiments on the magnetic effects of current.

where μ_0 is a constant called the permeability of the vacuum, its value being $\mu_0=4\pi\times10^{-7}$ webers/ampere metre.

The magnitude of the field is given by

$$dB=\mu_0 \frac{Idl\sin\theta}{4\pi r^2} \quad .. (13.20)$$

where θ is the angle between the vector dl and r, r being the position vector of P relative to the element.

The direction of dB is that of vector $dl\times r$. In the figure 13 13, dB at the point P is directed into the page at right angle to the plane of the figure. It is easy to make use of Biot-Savart law to get the field in case of regular geometry of current distributions.

Field at the centre of a circular coil

Consider a loop of wire of radius r carrying a current I, as shown in Fig 13.14 The loop

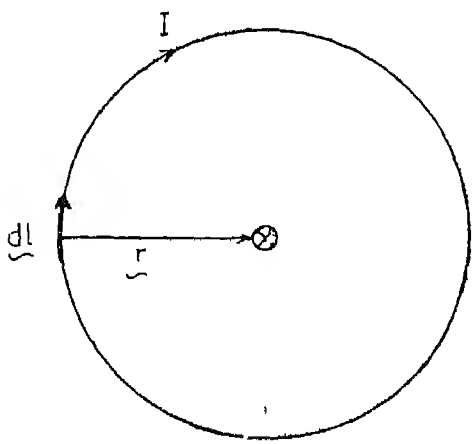


Fig. 13 14 Magnetic field at the centre of a circular current

lies in the plane of paper. We wish to calculate the magnetic field at the centre of the loop.

Consider the element dl of the loop. The magnetic field dB at the centre O due to this current element is given by Biot-Savart law,

$$dB = \frac{\mu_0}{4\pi} \frac{I.\ dl\times r}{r^3}$$

The direction of the field is perpendicular to the plane of the loop and points into the plane of paper. It is shown by the sign ⊗ Since the angle between dl and r is 90°, the magnitude of dB, using equation (13 20) is given by

$$dB = \frac{\mu_0 Idl}{4\pi r^2} \quad (13.21)$$

If we divide the loop into a large number of such elements we find that the field at O due to each element points into the plane of paper and its magnitude is given by equation (13 21). The total field at O is given by

$$B = \int dB = \int \frac{\mu_0 Idl}{4\pi r^2} = \frac{\mu_0}{4\pi}\frac{I}{r^2}\int dl$$

Now, dl is just the total length of the loop wire given by $2\pi r$. Hence

$$B = \frac{\mu_0 I}{4\pi r^2}\ 2\pi r = \frac{\mu_0 I}{2r}$$

If instead of a single loop, there is a coil of n turns, all wound over one another, then

$$B = \frac{\mu_0 In}{2r} \quad ... (13.22)$$

Field on the axis of the circular coil

The field at a point on the axis of the coil at a distance x from its centre can be shown to be

$$B = \frac{\mu_0 nIr^2}{2(r^2+x^2)^{3/2}} \quad ... (13.23)$$

The magnetic field lines due to a circular coil carrying current are shown in Fig. 13.15.

Field due to a solenoid

In Fig. 13 16 are shown the magnetic field lines due to a cylinderical coil carrying a current. Such a coil is normally known as a solenoid.

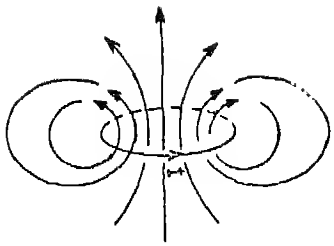


Fig. 13 15 Magnetic field lines due to a circular current

The field is fairly uniform in the interior of the coil and is given by

$$B=\mu_0 In \qquad ... (13.24)$$

where I is the current and n the number of turns/unit length

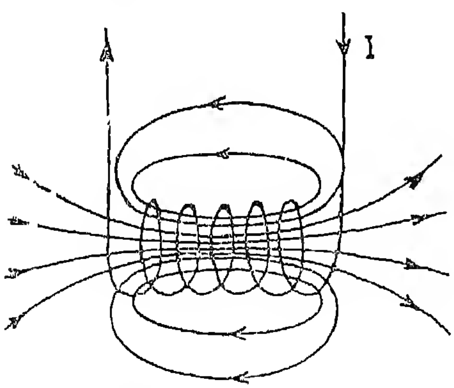


Fig. 13.16 Magnetic field lines due to a solenoid

Field due to a straight current

Fig. 13.17 shows the field lines due to a current I flowing in a straight wire The field is symmetrical about the current and its value at a point, distant r from the current is given by

$$B= \frac{\mu_0 I}{2\pi r} \qquad ... (13.25)$$

The field lines are circular and lie in a plane perpendicular to the current.

How to find out field direction ?

The direction of magnetic field in the above

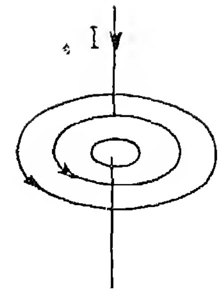


Fig. 13.17 Magnetic field lines due to a straight current

cases can be found out with the help of the following rules :

(i) *For circular current* · Curl the fingers of the right hand in the direction of the current, the stretched thumb then points in the direction of the field.

(ii) *For straight current* Grasp the wire in the right hand with the thumb pointing in the direction of current, the field lines then follow the direction of curled fingers around the current

Example 13.6

Calculate the magnetic field due to a circular coil of 200 turns, radius 0.1m, carrying a current 5A, (i) at a point on the axis of coil distance of point being 0.20m from the centre of coil (ii) at the centre of the coil.

Solution

(i) $$B = \frac{\mu_0}{2} \frac{nr^2 I}{(r^2+x^2)^{3/2}}$$

$\mu_0=4\pi\times 10^{-7}$, $n=200$, $r=0.1m$, $x=0.2m$, $I=2A$. Hence

(i) $$B=\frac{4\pi x 10^{-7}\times 200\times 0.1\times 0.1\times 5}{2\times (0.1^2+0.2^2)^{3/2}}$$

$$=5.62\times 10^{-4} \text{ tesla.}$$

(ii) $$B=\frac{\mu_0 nI}{2r}=\frac{4\times 10^{-7}\times 200\times 5}{2\times 0.1}$$

$$=62.8\times 1^{-4} \text{ tesla.}$$

13.9 Motion of a Charged Particle in a Uniform Magnetic Field

Fig. 13.18 shows a charged particle of mass m and charge q moving perpendicular to a uniform magnetic field **B**. It experiences a force $\mathbf{F}=q\,(\mathbf{v}\times\mathbf{B})$, where v is the velocity of the particle. Since $\mathbf{F}\perp\mathbf{v}$, the magnetic force will not speed up or slow down the particle, but will only change the direction of its motion. The particle will move with a constant speed

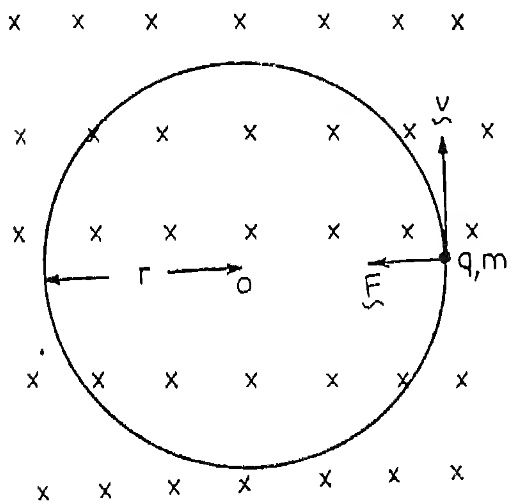


Fig. 13.18 Motion of a charged particle in a uniform magnetic field

but its path will be continually deflected in a way so as to form a circular trajectory.

The force $F=qvB$ gives rise to an acceleration $a=qvB/m$, which is constant in magnitude and perpendicular to velocity. This is the centrepetal acceleration needed to keep the particle in circular motion. Thus

$$\frac{qvB}{m}=\frac{v^2}{r} \qquad \dots (13\ 26)$$

where r is the radius of the circle. It may be noted that the circle lies in the plane perpendicular to **B**.

EXAMPLE 13 7

Electrons moving at right angles to a uniform magnetic field complete a circular orbit in 10^{-9} seconds What is the magnitude of the magnetic field ?

Solution

Time for completing one orbit is given by

$$t=2\pi r/v$$

Using eq (13 26), $t=\dfrac{2\pi r}{v}=\dfrac{2m}{qB}$

or $B=2\pi\ m/qt$

Now $m=9.1\times10^{-31}$kg., $q=1\ 6\times10^{-19}$C, $t=10^{-9}$s Hence

$$B=\frac{2\pi\times9\ 1\times10^{-31}}{1\ 6\times10^{-19}\times10^{-9}}$$

$=3\ 6\times10^{-2}$ tesla.

13.10 Force on a Current Carrying Conductor

When a current carrying conductor is placed in a magnetic field, the force experienced by the free electrons moving in the conductor is transmitted to the conductor as a whole Let a current I flow through a conductor AB placed in a uniform magnetic field B (Fig. 13 19). The magnitude of current is given by

$$I=enAv_d$$

where n is the density of free electrons, v_d their drift velocity and A the area of cross section of the conductor.

Now, each free-electron experiences a force $(-e)\,(\mathbf{v}_d\times\mathbf{B})$. In a length dl, the number of free electrons is nAdl. The total force experienced ~~by the element dl is, therefore, given~~ by

$$d\mathbf{F}=nAdl\,(-e)\,(\mathbf{v}_d\times\mathbf{B})$$

$$=-nAe(dl\,\mathbf{v}_d\times\mathbf{B})$$

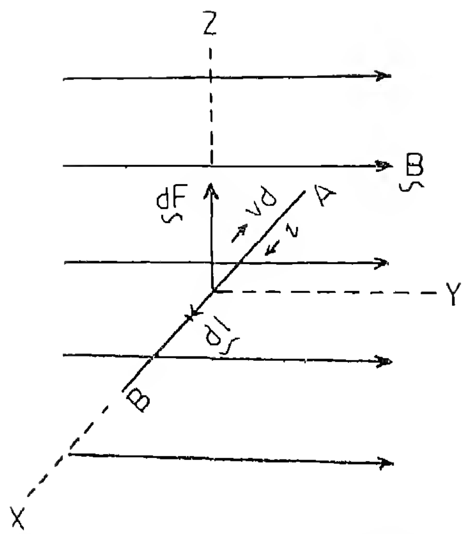


Fig. 13.19 Force on a current carrying conductor

If we represent the element of length by vector dl pointing in the direction of current, then v_d and dl point in opposite directions and the term dlv_d may be replaced by $-v_d dl$. Hence

$$\begin{aligned} dF &= -nAe\,(-vd\ dl \times B) \\ &= nAev_d\ (dl \times B) \\ &= I\,(dl \times B) \qquad \text{.. (13.27)} \end{aligned}$$

If **B** is uniform over the whole length l of the conductor, the force experienced by the conductor will be

$$\mathbf{F} = I\,(l \times \mathbf{B}) \qquad \text{... (13.28)}$$

The direction of the force is given by the vector $l \times \mathbf{B}$ where l vector is taken in the direction of the current. If the conductor is held at right angles to the field as in Fig. 13.19, the magnitude of force is given by

$$F = I\ lB \text{ newtons} \qquad \text{... (13.29)}$$

where I is in amperes, l in metres and B in tesla. The force is perpendicular to both, the conductor and the field. The relationship between the three vectors **F**, **I** and **B** may be described by the so called left-hand rule : Hold the thumb and the first two figures of the left hand mutually at right angles. Point the fore-finger in the direction of the magnetic field, centre finger towards current in the conductor, then the thumb indicates the thurst (force) on the conductor.

Force between parallel wires carrying current

Let AB and CD be two long parallel wires and P a point on AB (Fig. 13 20). The magnetic field at P caused by current I_2 in CD, is

$$B = \frac{\mu_o I_2}{2\pi r} \text{tesla}$$

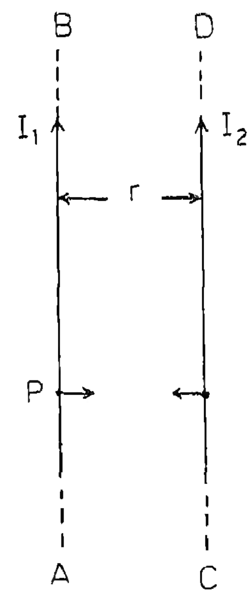


Fig. 13.20 Calculating the force between parallel currents

The field B is directed out of the page and is at right angles to AB. The force per metre at P is

$$F = BI = \frac{\mu_o I_1 I_2}{2\pi r} \quad \text{newton/metre.}$$

The force is at right angle to AB, towards CD, in the place of paper. The same formula will be obtained for the force exerted on CD The force will be that of repulsion if the currents are anti-parallel

Definition of ampere

When $I_1=I_2 = 1$ ampere and $r = 1$ metre

$$F = \frac{\mu_0}{2\pi} = 2\times 10^{-7} \text{ newton}$$

This is the basis of the definition of ampere. The ampere is the strength of that current which when flowing in two parallel infinitely long conductors of negligible cross section placed in vacuum at a distance of 1 metre from each other, produces between those two conductors a force of 2×10^{-7} newton per metre length.

EXAMPLE 13 8

Two long parallel wires carry currents of 3A and 4 A respectively in opposite directions. If the separation between them is 10 cm, find the force exerted by one over the other.

Solution

It can be readily seen that the force is that of mutual repulsion.

The magnitude will be given by

$$F = I_1 I_2/2\pi r$$

Here, $I_1=3A$, $I_2=4A$, $r=0.1m$, $=4\times10^{-7}$

Hence, $F = \dfrac{4\times10^{-7}\times3\times4}{2\pi\times0.1} 2.4\times10^{-6}$ newton m^{-1}

13.11 Torque on a Coil

Consider a rectangular loop of wire, ABCD of length l and breadth b suspended freely in a uniform horizontal magnetic field (Fig. 13.2a). Let the plane of the loop be parallel to the field. If there flows, now, a current I in the loop as indicated, each of the vertical sides, AD and BC will experience a force F=IBl. By left hand rule, the force on AD will point into the plane of the paper and on BC it will point out of the plane of the paper. The top view of the transverse section of the loop is shown in (Fig. 13.21 b), where the sign ⊙ indicates the cross section of wire AD with current coming out of the paper and the sign ⊗ the cross section of wire BC with current going into the paper. The small arrows show the forces acting on these wires

These forces constitute a couple and exert a torque on the loop given by

$$\tau = IBl\times b = IBA \qquad (13.30)$$

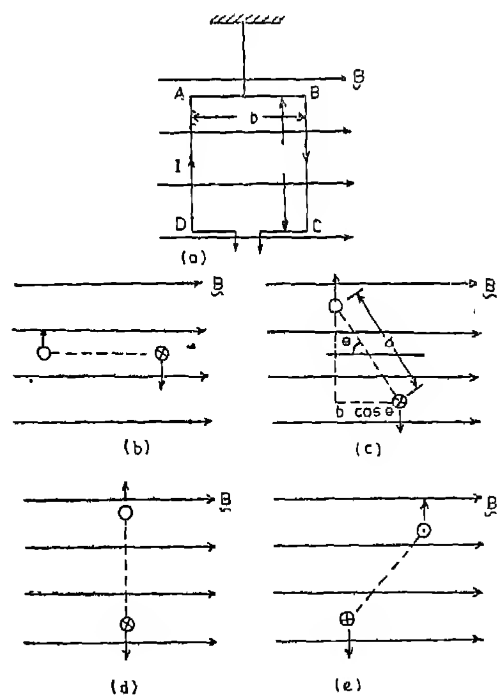


Fig. 13.21 Torque on a freely suspended coil in a uniform magnetic field

where A is the area of the loop. The loop would rotate. As it rotates, the value of torque

changes although the forces on its sides remain the same. When the plane of the loop makes an angle θ with the field (Fig 13.21 c) thet orque is given by

$$\tau = IBl \times b \cos\theta = IBA \cos\theta$$

If instead of a single loop of wire, there is a coil of n turns, the torque on the coil is given by

$$\tau = IBAn \cos\theta \qquad .. (13.31)$$

It can be easily seen that the torque reduces to zero value when the plane of the coil is perpendicular to the magnetic field (Fig. 13.21 d). Regarding the forces on the sides AB and CD, no forces act on them when the coil lies parallel to the field as the direction of current through them is parallel to the field

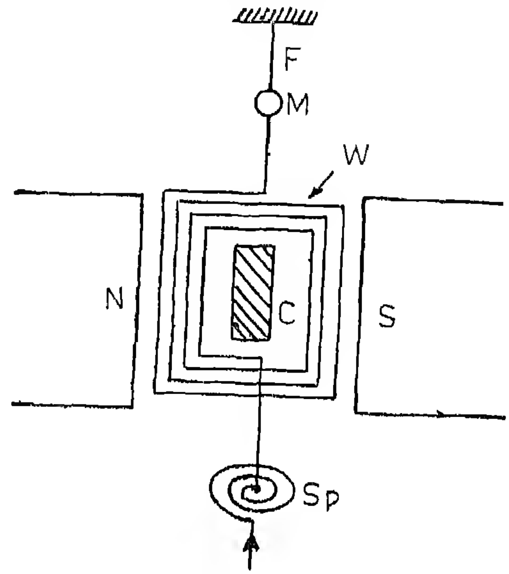
It can be shown that the expression (13.31) is valid for any geometrical shape of the coil-circular, rectangular etc , for a given area A of the coil

Equilibrium position of the coil

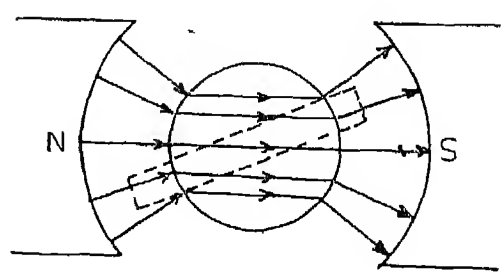
If there is no restraint on the coil, it would rotate until it is at right angles to the magnetic field. The torque in this position would be zero but because of its inertia of motion the coil will move past this position As soon as it crosses $\theta = 90^\circ$ position, a torque in the opposite direction develops (Fig. 13.21 e) which tends to bring it back. The coil would thus oscillate, back and forth, a few times, till its energy is dissipated away in overcoming frictional forces and it would ultimately come to rest at the position $\theta = 90^\circ$.

13.12 The Moving Coil Galvanometer

Consider the arrangement shown in Fig. 13.22. A coil made of many turns of fine copper wire wound over a metal frame is suspended by a torsion fibre in the magnetic field of a permanent magnet. A soft iron cylinder is mounted in the space between the cylindrically curved pole pieces. This makes the field lines radial (Fig. 13.22(b) The coil moves in the clearing between the poles and the cylinder. It can be easily seen that whatever



(a)



(b)

Fig 13.22 (a) Moving coil galvanometer. F-suspension fibre, M-mirror, W-coil, C-soft iron core, Sp-spring, NS-magnet
(b) Radial field between pole pieces

be the position of the coil the plane of the coil is always parallel to the field lines. Therefore, the deflecting torque when a current I passes through the coil, using equation (13.31) is given by ($\cos\theta = \cos 0 = 1$)

$$\tau = IBAn$$

where, n is the number of turns on the coil, A is the area of the coil and B is the magnetic field.

This torque will lead to a twist θ in the torsion fibre which sets up a restoring torque, $C\theta$, where C is the torsion constant. C is defined as the torque required to produce a unit twist in the fibre. In the equilibrium position the restoring torque equals the deflecting torque, hence

$$C\theta = IBAn$$

$$\text{or } \theta = \frac{BAn}{C} I \qquad \text{.. (13.32)}$$

Thus the deflection of the coil is directly proportional to the current in the coil.

The device described above is the basic current measuring device, called the galvanometer. The rotation of the coil is noted by reflection of a narrow beam of light from the small mirror attached to the coil. The suspended coil type instrument is delicate and requires careful handling. Very often a pivoted-coil type (Fig 13.23) is used which is rugged and portable though less sensitive. The basic principle remains the same.

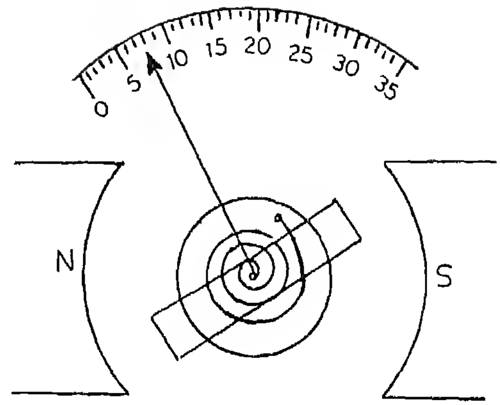


Fig. 13.23 Pivoted-coil type galvanometer

Ammeter

Any instrument used to measure currents is known as an ammeter. The galvanometer described above is a very sensitive ammeter. The pivoted-coil type can measure currents of the order of a microampere. However, the range of measurement of current of a galvanometer is quite small. For example, pivoted coil type, on full scale deflection can measure currents upto a few milliamperes only. It can, however be adopted for measuring large currents in the following manner.

Suppose it is required to measure a current I with the help of a galvanometer which produces a full scale deflection with a current i_g. Let the resistance of galvanometer coil be i_g. A low resistance R_s whose value is small compared with R_g is placed in parallel with the galvanometer as shown in Fig 13 24. This ensures that most of the current passes through

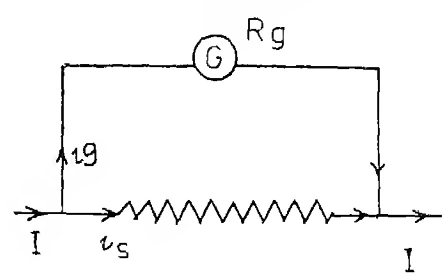


Fig. 13.24 Conversion of a galvanometer into an ammeter

the shunt while the current through the galvanometer does not exceed the safe limit i_g. The current I in the main circuit divides into

two currents, i_g through the galvanometer and i_s through the shunt, then

$$i_g R_g = i_s R_s$$

$$\text{or} \quad R_s = \frac{i_g}{i_s} R_g = \frac{i_g}{I - i_g} R_g \qquad \dots (13.33)$$

The following example may be illustrative.

EXAMPLE 13.9

A galvanometer whose resistance is 120 ohm has full scale deflection with a current of 0 0005A. Find the value of shunt that must be connected in parallel so that it can read a maximum current of 5A. What is the resistance of the ammeter so constructed ?

Solution

$I = 5A$, $R_g = 120$ ohm

$i_g = 0\ 0005A$

$$R_s = \frac{i_g}{I - i_g} R_g = \frac{0.0005}{4\ 9995} 120 = 0.012 \text{ ohm}$$

The resistance of the ammeter is that of the combination R_s and R_g in parallel. It is given by

$$R = \frac{R_s R_g}{R_s + R_g} = \frac{0.012 \times 120}{120 + 0\ 012} = 0\ 0120 \text{ ohm}$$

A galvanometer, may, therefore be converted into an ammeter to read any current. It is essential that the resistance of the ammeter be very small so that when it is inserted in a circuit it may not alter the value of current it is supposed to measure.

Voltmeter

An instrument used to measure potential difference is known as a voltmeter. A galvanometer can function as a voltmeter However, the range of measurement of a basic instrument is very small. The maximum potential difference that it can measure is that which will produce the full scale deflection in it and will be given by $V_g = R_g i_g$. Thus, in the above example, the galvanometer can measure a maximum potential difference of $V_g = 120 \times 0\ 0005 = 0\ 06$ volt only. It can, however, be adapted to measure higher potential differences. For this, it is necessary to connect a high resistance r in series with the galvanometer so that when a potential difference V ($>V_g$) is applied across the combination (Fig. 13.25)

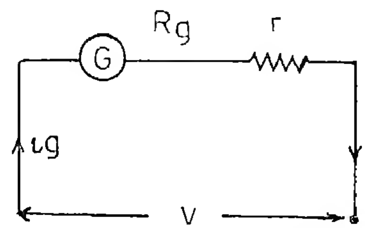


Fig 13.25 Conversion of a galvanometer into a voltmeter

the current through the galvanometer does not exceed the safe limit i_g. Thus for full scale deflection.

$$V = (R_g + r)\, i_g \qquad \dots (13.34)$$

The following example may be illustrative.

EXAMPLE 13.10

Find the value of resistance that must be connected in series with the galvanometer of example 13.9 so that it can measure a maximum potential difference of 6 volts.

Solution

The potential difference of 6 volts when applied across the combination of R_g and r, should produce full scale deflection in the galvanometer Hence

$6 = (R_g + r)\, i_g = (120 + r)\ 0.0005$

or, $r = 11880$ ohm

When a voltmeter is used to measure a potential difference V across a resistance R through which a current I is flowing it is desirable that the resistance of the voltmeter should

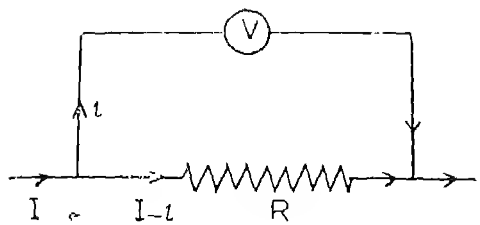


Fig. 13.26 The current through the voltmeter affects the measured value of voltage

far exceed the value of R The situation is shown in Fig. 13 26. When a voltmeter is connected across R, it draws a current, say, i, leaving a current $I-i$ to flow through R. The potential difference across R now change from $V=R\,i$ to $V'=R(I-i)$ It is V', instead of V, which is measured by the voltmeter. Hence in order that V may stay close to the desired value V, i should be very small i.e. the voltmeter must have a very high resistance.

Exercises

13.1 Is any work done by a magnetic field on a moving charge ?

13.2 Assuming that the earth's magnetic field is due to a large circular loop of current in the interior of the earth, what is the plane of the loop and what is the direction of current around it ?

13.3 A small coil carrying a current is placed in a uniform magnetic field. How does the coil tend to orient itself relative to the magnetic field ?

13 4 Explain why the two parallel wires carrying current in opposite direction repel each other.

13 5 Two parallel wires carrying current in the same direction attract each other while two beams of electrons travelling in the same direction repel each other Explain why ?
(Hint : A current in the wire produces only a magnetic field while an electron beam is a source of both, an electric and a magnetic field)

13.6 What is the number of free electrons in a piece of silver of area of cross section $1.0\times10^{-4}m^2$ and length 1m. Atomic weight of silver=108, density of silver=105 kg/m^3. Assume there is one free electron per atom.
(5.83×10^{24})

13.7 How many electrons pass through a lamp in one minute if the current is 300 mA. (1.12×10^{20})

13.8 A potential difference V is applied to a conductor of length L, diameter D. How are the electric field E, the drift velocity v_d and the resistance R affected when (i) V is doubled, (ii) L is doubled, (iii) D is doubled.

13.9 A platinum wire has resistance of 10 ohms at 0°C and 20 ohms at 273°C. Find the value of temperature coefficient of resistance. ($\alpha = 1/273$ per °K)

13.10 At Rs 0.40 per kWhr, how much will it cost to have a 60W lamp burning for 5 days ? (Rs. 2 88)

13.11 Near room temperature, the thermo e.m.f. of a copper constantan couple is $40\mu V$ per degree What is the smallest temperature difference that can be detected with a single such couple and a galvanometer of 100 ohm resistance capable of detecting currents as low as 10^{-6} amperes. (2 5°)

13.12 The electron in the hydrogen atom circles around the proton with a speed of 2.18×10^6m/s in an orbit of radius 5.3×10^{-11} m. What magnetic field does it produce at the proton. (12.5 tesla)

13.13 A horizontal wire 0.10m long carries a current of 5A. Find the magnitude and direction of the magnetic field which can support the weight of the wire, assuming its mass is 3.0×10^{-3}kg/m. ($5\ 88 \times 10^{-2}$ tesla)

13.14 A long straight wire carries a current of 2A. An electron travels with a velocity of 4.0×10^4m/s parallel to the wire 0 1m from it, and in a direction opposite to the current. What force does the magnetic field of current exert on the moving electron ? (2.56×10^{-20} newton)

13.15 The coil of a galvanometer is 0.02×0.08 m. It consists of 200 turns of fine wire and is in a magnetic field of 0.20 tesla. The restoring torque constant of the suspension fibre is 10^{-6} Nm/degree. Assuming the magnetic field to be radial,

(i) What is the maximum current that can be measured by this galvanometer if the scale can accommodate 45° deflection ?

(ii) What is the smallest current that can be detected if the minimum observable deflection is 0.10 degree ? (7×10^{-4} A, 1.6×10^{-6} A)

13.16 A galvanometer has an internal resistance of 1.0 ohm. It gives maximum deflection for a current of 50mA. Show how this instrument can be converted into (i) a voltmeter with a maximum reading of 2.5 volt (ii) an ammeter with a maximum reading of 2.5A.

(series resitance=49 ohm,
shunt resistance=1/49 ohm)

UNIT 14

Electromagnetic Induction

Earlier we have seen that electric currents generate magnetic fields In this chapter we shall deal with the converse effect the production of electric currents with the help of magnetic fields The phenomenon is known as electromagnetic induction. It was discovered by Faraday in 1831.

Faraday described the phenomena in terms of magnetic flux, which we define first

14.1 Magnetic Flux

Consider a closed curve, Fig. 14 1, bounding a plane surface of area A. Suppose there

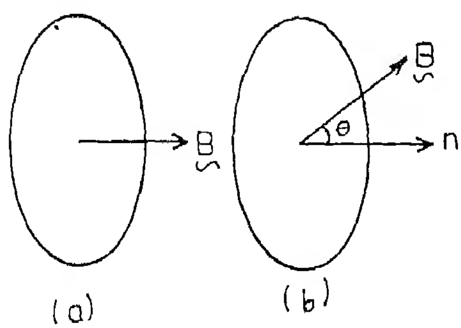


Fig. 14.1 Magnetic flux through an area depends on its orientation w r t. the magnetic field

is a uniform magnetic field B perpendicular to the surface. We define the magnetic flux through the area A as

$$\phi = \mathbf{AB} \qquad 1(4.1)$$

If B is not perpendicular to the surface, the flux through A is given by

$$\phi = \mathbf{B}\,\mathbf{A}$$
$$= \mathrm{BA}\cos\theta \qquad (14\ 2)$$

where θ is the angle which the vector **B** makes with the normal to the surface.

In S I system, unit of magnetic flux is weber. A uniform magnetic field of 1 tesla, normal to an area of 1 m^2 would give rise to a flux of 1 weber.

Since $B = \phi/A$, field B is quite often expressed as weber/m^2 instead of tesla.

Positive and negative flux

A normal can be drawn to a plane in two ways. The flux is taken as positive when the normal points out in the direction of the field, i e $\theta=0$ When the normal points in the opposite direction, $\theta=180°$ and flux=BA cos 180° $=-$BA, that is, negative.

14 2 Laws of Electromagnetic Induction

There are two laws governing the phenomenon of electromagnetic induction. (*i*) Faraday's law, that gives us the magnitude of induced e m f. (*ii*) Lenz's law which tells us the direction in which an induced e m.f. acts.

(*i*) *Faraday's Law*. Faraday showed that an e.m f. is induced in a circuit whenever there is a change in the magnetic flux enclosed by a circuit. The magnitude of the induced e.m.f is equal to the rate of change of flux through the circuit.

Thus

$$|E| = \frac{d\phi}{dt} \qquad . \ (14.3)$$

(*ii*) *Lenz's Law*: The polarity (or sense) of an induced e m.f. is such that the induced current flows in a direction so as to oppose the change which is causing the induction. Thus, if the magnetic flux is decreasing, the induced current in the circuit must flow in a direction to create flux in the direction of original flux. And, if the flux is increasing the induced current would create a flux that will be opposite in direction to the original magnetic flux.

Remembering Lenz's law, Faraday's law is written as

$$E = -\frac{d\phi}{dt} \qquad . \ (14.4)$$

When the equation is applied to a coil of N turns, an e m f. develops in every turn and the total e m f. that appears at the terminals of the coil is the sum of all these e.m f.s In such a case, induced e.m.f. is given by

$$E = -N\frac{d\phi}{dt} \qquad \ldots (14.5)$$

If flux changes from ϕ_1 to ϕ_2 in time t, then the average e m.f induced is given by

$$E = -N\frac{(\phi_2-\phi_1)}{t} \qquad . \ (14.6a)$$

If ϕ is in webers and t in seconds, then E is in volts.

Example 14.1

A 100 turn close-packed coil of diameter 0.20 m is placed with its plane perpendicular to a uniform magnetic field. The field value, then varies at a uniform rate from 0.10 weber/m² to 0 30 weber/m² in 5.0×10^{-2}s. Find the e m f. induced in the coil

Solution

$$E = -N\frac{(\phi_2-\phi_1)}{t}$$

Here $N = 100$

$$\phi_2-\phi_1 = A\,(B_2-B_1) = \pi \times 0.10^2\,(0\ 30-0.10) \text{ webers.}$$

$$t = 5.0\times10^{-2}\text{ s,}$$

$$E = -\frac{100\times0.10^2\times0\ 20\pi}{5\times10^{-2}}$$

$$= -12.6 \text{ volts}$$

Minus sign indicates that the e m.f. acts in such a direction as to send a current which would oppose the increase in flux.

Example 14.2

A coil lies in a uniform magnetic field B with its plane perpendicular to the field, Fig 14.2(a). In this position the normal to the coil makes an angle 0° with the field The coil rotates at a uniform rate to complete one rotation in time T. Find the average induced e m f. in the coil during the interval when the coil rotates,

(*i*) from 0° to 90° position, Fig. 14.2(b),
(*ii*) from 90° to 180° position, Fig. 14.2(c),
(*iii*) from 180° to 270° position, Fig. 14.2(d),
(*iv*) from 270° to 360° position, Fig. 14.2(a).

Solution

(*i*) initial flux in position (a), $\phi_1 = B\times A$
final flux in position (b),

$$\phi_2 = BA\cos 90° = 0$$

$$\text{time } t = \frac{T}{4}$$

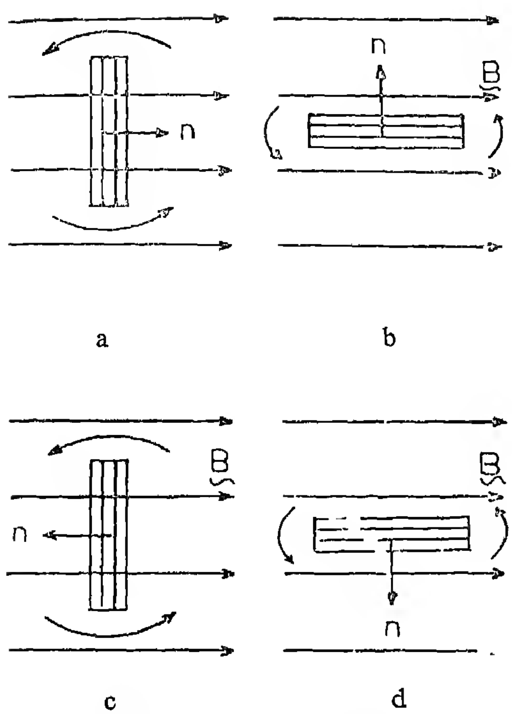


Fig. 14.2 Illustrates example 14.2

Average induced e.m.f.

$$E = -(\phi_2 - \phi_1)/t$$
$$= -\frac{(0 - BA)}{T/4} = \frac{4BA}{T}$$

(*ii*) initial flux, $\phi_1 = 0$

final flux, $\phi_2 = B \times A \cos 180° = -BA$

Average induced e.m f.

$$E = -\frac{(-BA - 0)}{T/4} = \frac{4BA}{T}$$

(*iii*) initial flux, $\phi_1 = -BA$

final flux, $\phi_2 = 0$

Average induced e.m.f.

$$E = -\frac{0-(-BA)}{T/4}$$
$$= \frac{-4BA}{T}$$

(*iv*) initial flux, $\phi_1 = 0$

final flux, $\phi_2 = BA$

Average induced e m f.

$$E = -\frac{(BA - O)}{T/4} = \frac{-4BA}{T}$$

Since the sense of the induced e.m f. in the second half rotation is opposite to that in the first half rotation, the induced current will change its direction after first half rotation.

14 3 Methods of Creating Induced E M.F.

An induced e.m.f. is produced by the change in magnetic flux passing through a circuit. The magnetic flux may be changed, in view of equation (14 2), by one of the following methods :

(*a*) changing the magnetic field B

(*b*) changing the area A of the loop (circuit)

(*c*) changing the relative orientation of B and A.

Induced e m.f. by changing B

This can be accomplished in following ways.

(*i*) In Fig. 14.3 an induced e m.f. will be set up in the wire loop when there is a relative motion between the loop and the magnet. By keeping one of them, the loop or the magnet, fixed and moving the other one changes the magnetic field at the loop This causes a change in the flux linked with the loop which produces the induced e.m.f. in the circuit. The direction of induced current follows Lenz's law. Thus, in a given situation the induced current flows in the indicated direction so as to oppose the change in magnetic flux through the loop caused by the relative motion. Replacing the magnet with a coil carrying current would give us the same results.

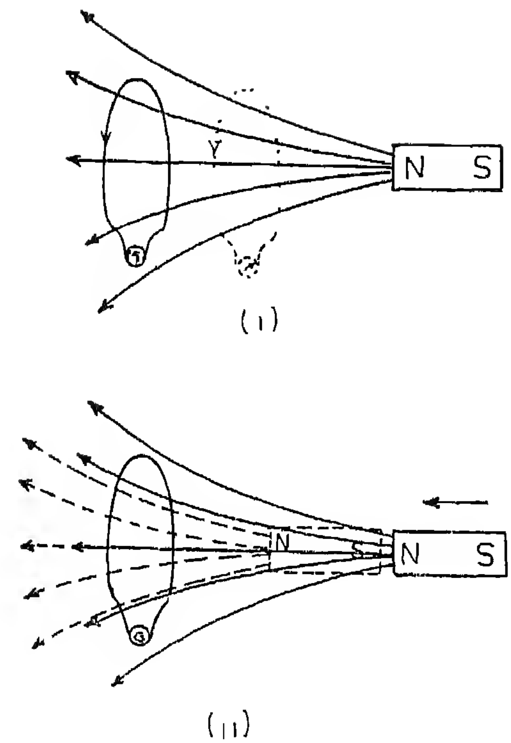


Fig. 14 3 Induced e.m f is caused in the loop by change in flux due to change in B at the loop by relative motion of the loop and the source of the magnetic field

(*ii*) Another way of changing B is illustrated in Fig 14 4 A change in the current in the circuit B produces a corresponding change in the magnetic field at the site of a neighbouring circuit A The consequent change in the magnetic flux through the loop A causes an induced current to flow through it The effect is more pronounced when the current in the loop B is switched on or off than when it is varied by sliding the contact in the variable resistor R. The reason is, the magnitude of the induced e.m.f. depends upon the rate of change of flux,

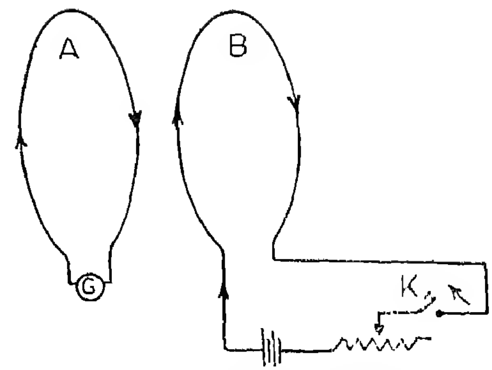


Fig. 14.4 Induced e.m.f. caused by change in the magnetic flux due to changing current in the neighbouring circuit

the faster the rate of change, the greater is the induced e m f. and the current caused by it.

Induced e m f by changing A

Consider a conductor ab moving with a velocity v towards right on U shaped conducting rails situated in a magnetic field, as shown in Fig 14 5. The field is uniform and points

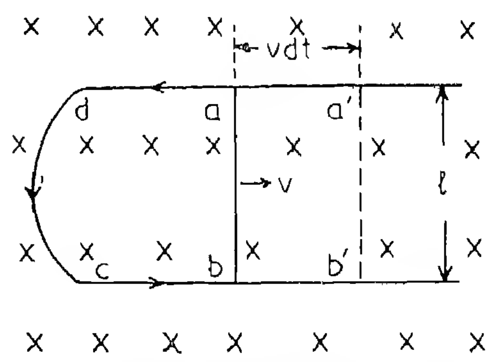


Fig 14 5 Induced e m f. caused by change in flux due to change in area of the circuit

in the plane of the paper. As the conductor slides, the area of the circuit changes from abcd to a′ b′ c d in time dt This causes an

increase in flux given by

$$d\phi = B\,(\text{area } a'\,b'\,c\,d - \text{area } abcd)$$
$$= B \ \text{area } a'\,b'\,b\,a$$
$$= B\,l\,vdt \quad \ldots (14.7)$$

This gives rise to an induced e.m f. in the circuit whose magnitude is given by

$$|E| = \frac{d\phi}{dt} = Blv \quad . \ (14.8)$$

The induced current flows in the direction shown according to Lenz's law.

EXAMPLE 14.3

An aeroplane with a wingspan of 30 metres flies at a horizontal speed of 100 metres per second in a region where the vertical component of the magnetic field due to earth is 5.0×10^{-5} weber/m². What is the potential difference between the tips of the wings ?

Solution

The metal between the wing tips can be considered to be a single conductor which is moving at right angles to a magnetic field The induced e m f. creates a potential difference between the tips Its value is given by

$$E = Blv$$

Here

$B = 5.0 \times 10^{-5}$ weber/m², $l = 30$ metre, $v = 100$m/s.

Hence,

$$E = 5.0 \times 10^{-5} \times 30 \times 100$$
$$= 0.15 \text{ volt}$$

Induced e.m f. by changing relative orientation of the coil and the field

Consider a coil free to rotate about an axis in its own plane, the axis being perpendicular to the magnetic field, as in Fig. 14.6. The flux through the coil when its normal makes an

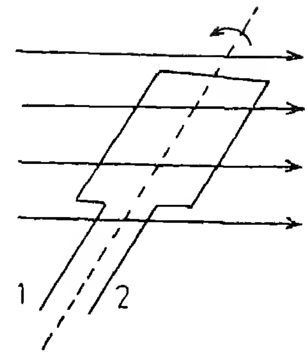


Fig. 14.6 Rotating coil in a magnetic field

angle θ with the field is given by

$$\phi = BA\cos\theta \quad (14.9)$$

As the coil rotates, the magnetic flux through it changes continually. Since the flux is changing there will be an e.m f. induced in the coil. The direction of the e.m.f. will, however, not remain constant but go on reversing after every half rotation (Example 14 2). The outer terminals 1 and 2 of the coil will alternately become +ve and −ve after intervals of time $\frac{T}{2}$ where T is the period of rotation. If these terminals are connected to an outside circuit, a current will flow in it which will reverse its direction at intervals of time $\frac{T}{2}$.

Let the angular velocity of the coil be ω. If we measure time t from the instant when the coil was perpendicular to the field *i.e.* $\theta = 0$ at $t = 0$ then at time t $\theta = \omega t$

Therefore, from equation (14.9),

$$\phi = BA\cos\omega t$$

and

$$\frac{d\phi}{dt} = -BA\omega\sin\omega t \quad . \ (14.10)$$

If the coil consists of N turns, the induced e.m.f is given by

$$E = -N \frac{d\phi}{dt} = BA\,\omega\,N \sin \omega t \quad . \ (14.11)$$

The induced e.m.f is an alternating one. The e.m.f is maximum when $\omega t = 90°$, *i.e.* when the plane of the coil is parallel to the field Denoting the maximum value by E_0 we have,

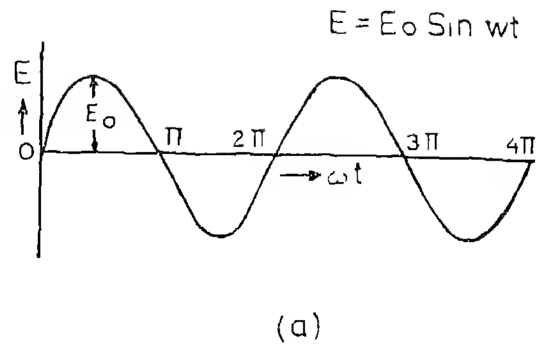
$$E_0 = BA\omega N \quad . \ (14.12)$$

The value of E_0 depends upon the strength of the field, the area of the coil, the speed of rotation and the number of turns in the coil, E_0 is called the amplitude or peak value of e.m.f. If the coil makes f rotations per second, we have

$$E = E_0 \sin \omega t = E_0 \sin 2\pi f t \quad .. \ (14.13)$$

If E is plotted against t, the graph is a sine curve, as shown in Fig 14.7(a) Such an e.m.f. is said to be sinusoidal We also refer to it as alternating e.m.f. This e.m.f will appear at the terminals of the rotating coil and may be transferred to an external circuit by suitable means.

It may be noted that had we measured the time since the instant when the coil was parallel to the field, *i.e.* at $t = 0$, $\theta = \pi/2$, then at time t we would have, $\phi = BA \cos(\omega t + \pi/2)$. It would have given us, $E = E_0 \cos \omega t$. The curve of e.m.f. would have been as shown in Fig. 14.7(b) Except for the matter of choosing the zero of time, there is no essential difference between the sine and cosine forms. Both are known as sinusoidal functions.



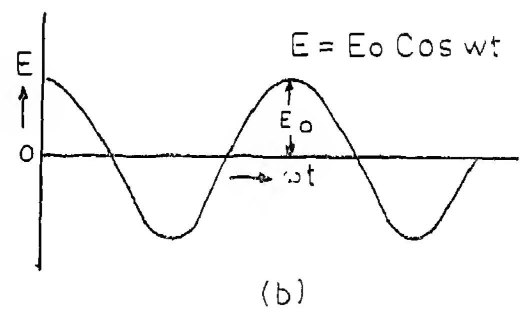


Fig. 14.7 Wave forms of alternating e.m.f.

EXAMPLE 14.4

A rectangular coil of dimensions $0.10\text{m} \times 0.05\text{m}$, consisting of 1000 turns rotates about an axis parallel to its long side, making 3000 revolutions per minute in a field of 100 gauss. What is the maximum e.m.f. induced in the coil, and the instantaneous value of e.m.f when the coil is at 45° to the field ?

Solution

3000 rev. per minute = 50 rev. per second

$$\omega = 2\pi \times 50 = 100\pi$$
$$E = \omega N A B \sin \omega t$$

Here,

$$\omega = 100\pi,\ N = 1000;\ A = 0.10 \times 0.05\text{m}^2;$$
$$B = 100 \times 10^{-4} \text{ tesla}.$$

$$E = 100\pi \times 1000 \times 0.10 \times 0.05 \times 100 \times 10^{-4} \sin 100\,\pi t$$

$$= 5\pi \sin 100\,\pi t$$

$\therefore$ Maximum value of induced e.m.f.,

$$E_0 = 5\pi = 15.7 \text{ volts}$$

When $\omega t = 45°$, $E = E_0 \sin 45°$

$$= 15.7 \times \sqrt{\tfrac{1}{2}} = 11.2 \text{ volts.}$$

14.4 The Generator or Dynamo

What we have described above is the principle of a generator, a means of producing electric current continually by mechanical means. It is the most important method of converting mechanical energy into electrical energy.

Fig. 14.8 shows schematically the essential parts of a generator. A coil abcd capable of

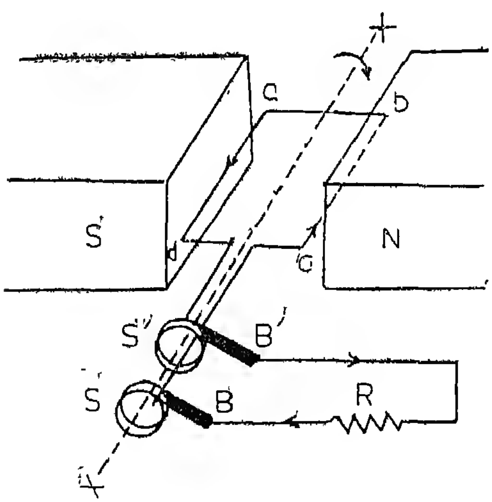


Fig. 14.8 A C. Generator

rotation about the axis XX is situated in the field of a magnet NS. As the coil rotates an alternating e.m.f. develops in the coil which is fed to the external circuit by means of a pair of metal slip rings, S and S′ which are fixed rigidly to the same shaft which is used to rotate the coil Each ring is connected to one terminal of the coil and rotates with the coil The rings maintain sliding contact with the brushes, B and B′. The output e.m f of the generator is alternating as shown in Fig 14 7. As the coil rotates, the current flows out through the brush B for one half of a revolution and through the brush B′ for the next half. The current through the external resistance R is also alternating.

If one directional, *i e.* direct current is desired, the slip rings are replaced by what is known as a split ring commutator as shown in Fig. 14.9 The commutator rotates with the coil. As before brushes make sliding contact

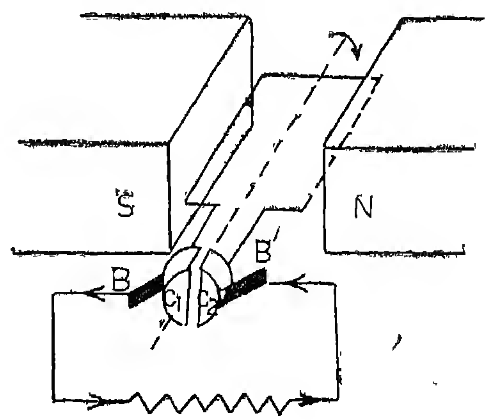


Fig. 14.9 D. C. Generator

with the commutator for half the rotation when C_1 is positive, it is in contact with the brush B, for the other half of rotation it is C_2 which is positive and is in contact with B. Thus the current always leaves the generator through the brush B and we get a direct current in the external circuit. The output, however, is pulsating as shown in Fig 14 10. If we have several coils, uniformally spaced, as in Fig. 14.11, all

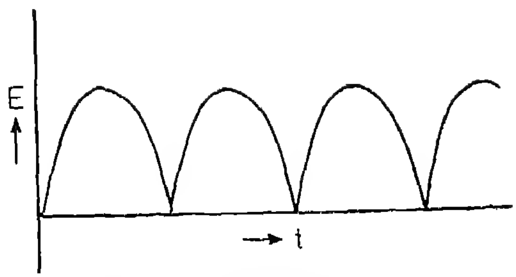


Fig. 14.10 Output e.m.f. from a single coil D. C. Generator

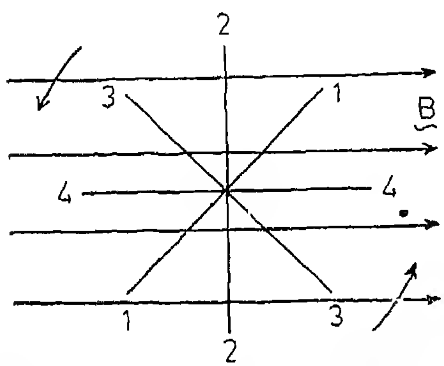


Fig. 14.11 Four coils rotating in a magnetic field

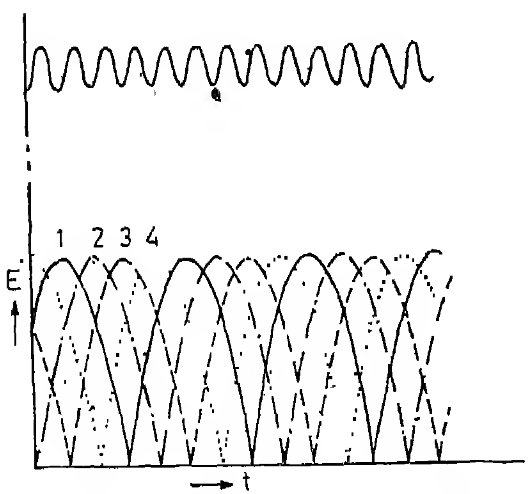


Fig. 14.12 Output e.m.f. from four coils rotating in a magnetic field

connected in series, the maximum value of e.m.f. occurs in each coil at different instants and the net effect is an almost constant unidirectional e.m.f. with a small ripple. Fig. 14.12 shows results achieved with four coils. More coils would give an even more smooth output.

14 5 Mutual Inductance

Consider two coils P and S placed near each other, Fig. 14.13(a). If coil P carries a current, it produces a magnetic field which produces a magnetic flux through S. If the current in the coil P, called the primary, is changed, it would cause an induced e.m.f. in the coil S, called the secondary.

Let the current through primary coil at any instant be I_1. Then the magnetic flux at any part of the secondary coil will be proportional to I_1 that is

$$\phi_2 \propto I_1$$

Therefore, the induced e.m.f. in the secondary, when I_1 changes, is given by

$$E = -\frac{d\phi_2}{dt}$$

i.e. $$E \propto -\frac{dI_1}{dt} = -\frac{MdI_1}{dt} \qquad \text{... (14.14)}$$

where M is the constant of proportionality, called the mutual inductance of the two coils. It is defined as the e.m.f. induced in the secondary by a unit rate of change of current in the primary. When E is expressed in volts, and $\frac{dI_1}{dt}$ in amperes/second, M is in volt/second/ampere, called henry (H). The mutual inductance between the circuits is 1H, if a rate of change of current of 1A/s in the primary produced an induced e.m.f. of 1 volt in the secondary.

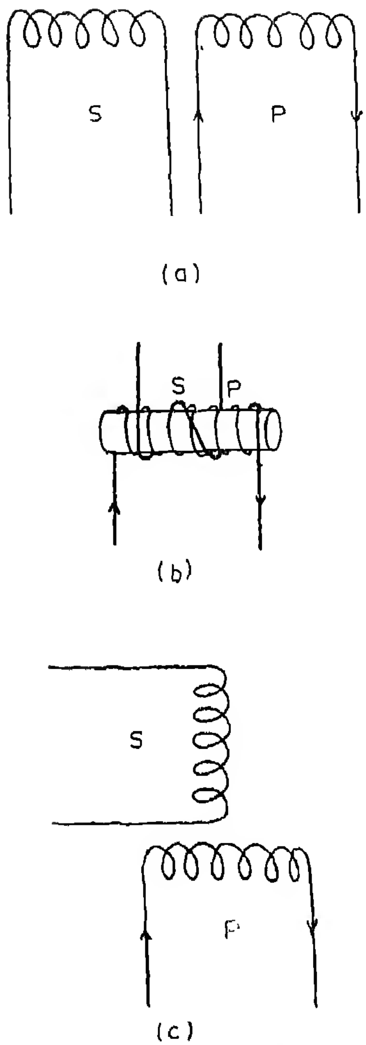


Fig 14.13 (a) Mutual induction. Variation of current in primary coil P induces an e.m.f. in the secondary coil S.
(b) case of largest value of M
(c) case of least value of M.

M depends upon the number of turns in the coils, their geometrical shape and their separation. It is maximum when the entire flux of the primary links with the secondary. Thus the value of M will be minimum when one coil is perpendicular to another as in Fig. 14.13(c). The mutual inductance is further increased, if the coils in Fig. 14 13(b) are wound over an iron core, by a factor μ, where μ is the permeability of iron.

14.6 Self-inductance

When a current flows in a coil, it gives rise to a magnetic flux through the coil itself. If the current strength changes, the flux changes and an e.m.f. is induced in the coil. This e.m.f. is called self-induced e m.f. and the phenomenon is known as self-induction.

It is easy to see that the flux through the coil is proportional to the current through it, *i. e.*

$$\phi \propto I$$

and, therefore, the induced e.m.f. E is given by

$$E = \frac{-d\phi}{dt}$$

$$i.e. \quad E \propto -\frac{dI}{dt} = -L\frac{dI}{dt} \qquad \dots (14.15)$$

where L is called the self-inductance of the coil. Like M, L is also measured in henry. A coil has self-inductance of 1 henry if an e.m.f. of 1 volt is produced in it when the current passing through it changes at the rate of one ampere per second.

Self-inductance, often called inductance, is a constant of the coil. It depends upon the number of turns, area of cross section, and the permeability of the core material. The larger the number of turns and area of cross section of a coil, the larger is its inductance. If coil is wound over an iron core, the self-inductance

increases by a factor μ where μ is the permeability of iron. A coil possessing an appreciable inductance is known as an inductor.

EXAMPLE 14.5

What e.m.f will be induced in a 10H inductor in which the current changes from 10 amperes to 7 amperes in 9.0×10^{-2} seconds ?

Solution

$$E = -L\frac{dI}{dt}$$

$$= -L\frac{(I_2 - I_1)}{t}$$

$$= -10\frac{(7-10)}{9.0\times10^{-2}}$$

$$= 333 \text{ volts}$$

Inductance in a d.c. circuit

Figure 14.14 shows an inductance L and a resistance R (including the resistance of the

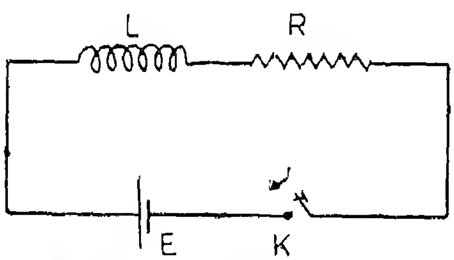


Fig. 14.14 Inductance in a d c. circuit

coil L) connected to a battery of e m.f. E. When the key K is closed, the current begins to grow. As the current increases, the magnetic field associated with it also increases and so does the magnetic flux through the coil. This increasing flux induces an e.m.f which produces a current opposed to the one which is growing, (Lenz's law). This limits the rate of rise of the current and it takes some time before the current reaches a steady value given by Ohm's law, $I_0=E/R$ The effect of inductance, therefore, is to increase the time taken by the current to reach its limiting value I_0 The length of this time depends on the value of L and may vary from a few milliseconds to several seconds. The manner in which I attains the value I_0 is shown in Fig. 14.15 for two values of L.

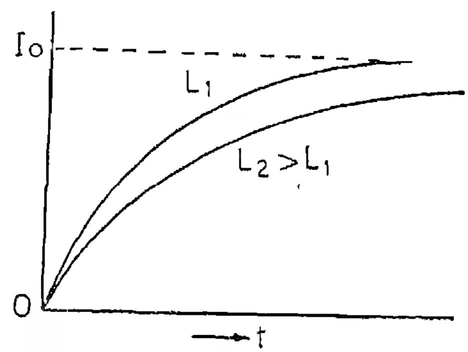


Fig. 14.15 Growth of current in an inductive circuit

Similarly, when a current flowing in a circuit containing inductance is interrupted, an induced e m.f. is set up in the circuit which tends to maintain the current. Thus in Fig. 14.16 if the switch K is opened, the current decreases rapidly, if this change takes place very rapidly, the induced e.m.f. may be quite

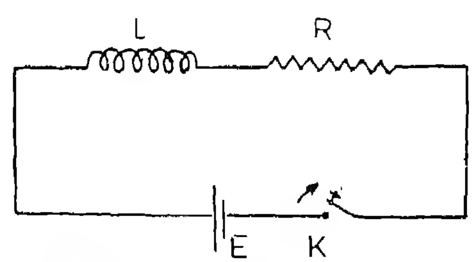


Fig. 14.16 Opening the switch in an inductive circuit leads to sparking

large and may cause a spark to jump across the partially open switch When the initial current is large and the inductance high, the spark may be sufficiently strong to cause damage to switch contacts and insulation. Switches for d.c. circuits, in such cases, are so designed as to allow the current to decay gradually and thus prevent sparking.

14.7 Some Phenomena Connected with Inductance

(i) Eddy currents

When a sheet of metal is placed in a changing magnetic field, induced currents are set up in the sheet which oppose these changes. These are known as eddy currents. The currents are circular, their sense of flow can be ascertained by applying Lenz's law. Fig. 14 17 shows some eddy currents in a metal sheet placed in an increasing magnetic field pointing in to the plane of paper.

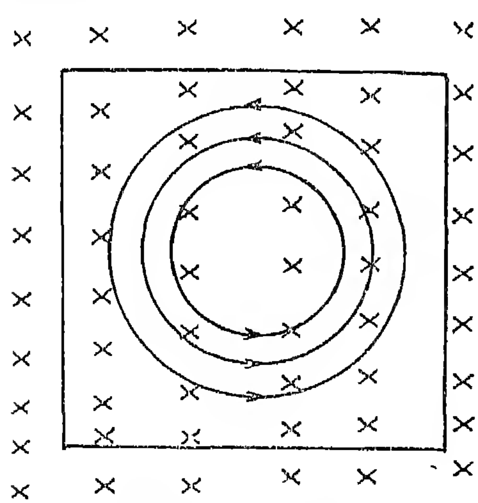


Fig. 14.17 Eddy currents

Since the resistance of metals is quite low, these currents may be quite large and may produce considerable heating effects. Eddy currents are used in heating small metal specimens by putting them in rapidly changing magnetic fields produced by high frequency alternating currents. It is called inducation heating.

Eddy currents are considered undesirable in electrical appliances and machinery, where iron is widely used, as they cause unnecessary heating and wastage of power. This is avoided by building up the desired iron part with thin sheets of the metal which are insulated from each other. The planes of sheets are placed perpendicular to the direction of currents that would be set up by the e.m.f induced in the material. The insulation between the sheets then offers high resistance to the induced e.m.f. and the eddy currents are substantially reduced

(ii) Electromagnetic damping

When a current is passed through a galvanometer the galvanometer coil usually suffers a few to and fro oscillations before settling down to its proper deflected position The motion of the coil is damped largely because of electromagnetic damping—as the coil moves in the magnetic field, a counter e m.f. is induced in the coil which opposes its motion. The electromagnetic damping can be further increased by winding the coil on a metallic frame. As the frame moves, eddy currents are generated in the frame which use up energy and hence damp the motion. In a properly constructed galvanometer oscillations can be prevented completely—the coil deflects and stays at its final position.

14.8 Alternating Current

Let a source of alternating e.m.f. (represented by the symbol (~) be connected across

a resistance R, Fig. 14.18. As the e.m.f., $E = E_0 \sin \omega t$, changes with time, the current

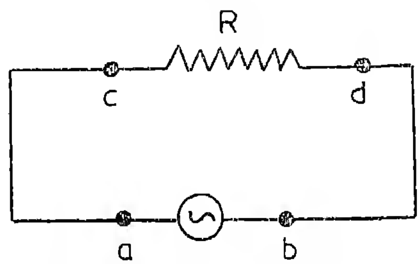


Fig. 14.18 A. C. source with resistance

through the circuit also changes. At any instant, the current would be given by Ohm's law

$$I = \frac{E}{R} = \frac{E_0}{R} \sin \omega t$$

$$= I_0 \sin \omega t = I_0 \sin 2\pi ft. \quad \ldots (14.16)$$

where $I_0 = E_0/R$ is the maximum or peak value of current, f is the frequency and t is the time. The current is sinusoidal and is known as alternating current. Its variation with time

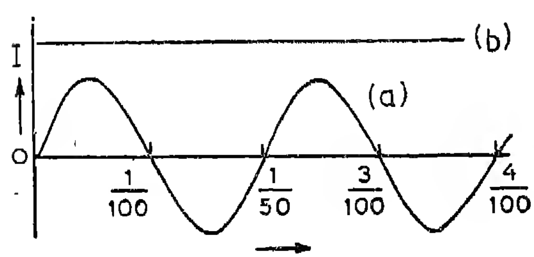


Fig. 14.19 (a) Wave form of alternating current
(b) steady current

is shown in Fig. 14.19, the time scale chosen is for the commonly used 50 c/s alternating current For half the cycle the current flows in one direction and for the other half in the opposite direction. A steady direct current is also shown in the same diagram for comparison.

Since an a.c. current* varies in magnitude continuously and changes direction periodically, the effects produced by it (such as the magnetic effect) will also vary with time. The question immediately arises as to how can we measure such a current ? How do we define an ampere of an alternating current ? There is only one effect which is independent of the direction of the electric current—the heating effect, which we make use of for this purpose. We compare the heating effect of an alternating current with that of a direct current and define the former in terms of the latter.

The effective value of an alternating current, I_{eff} is, therefore, defined as that magnitude of direct current which produces the same heating effect in a given resistance as the given alternating current. Thus an a.c. ampere would be an a c. current that produces a heating effect equal to that of one d.c. ampere.

It is found that an a.c. current $I = I_0 \sin \omega t$, produces the same heating effect as d.c. current of $I_0/\sqrt{2}$. The value

$$I_{eff} = \frac{I_0}{\sqrt{2}} = 0.707\ I_0 \quad \ldots (14.17)$$

is known as the effective value of the a.c. current. This is also known, sometimes, as root-mean-square or r.m.s. current.†

The effective or r.m s. value of an a c. voltage is defined in an exactly similar way and we have

$$E_{eff} = \frac{E_0}{\sqrt{2}} = 0.707\ E_0 \quad \ldots (14.18)$$

* Since a. c means alternating current, it may seem wrong to use the terms, an a c. current or an a c voltage, from language point of view, but it has now become a common practice to use these terms

† The reason is, the square root of the mean of the square of the current i e $\sqrt{(I^2)^{\frac{1}{2}}}$ av. $= I_0/\sqrt{2}$.

Hereafter, whenever numerical values of a.c. voltage and current are given, it is effective values which are meant unless specifically stated otherwise. Thus an a c. current of 10A means that the current varies in a sinusoidal manner with a peak value of $10\sqrt{2}$ A. An electric supply at 220v means the peak value of voltage is $220\sqrt{2} = 310$v.

14.9 A. C. Circuit Containing Resistance Only

Figure 14.18 shows a resistance R connected in series with an a.c. source of e.m.f.

$$E = E_0 \sin \omega t \qquad \text{... (14.19)}$$

At any instant, the potential difference across the resistance R, between the points c and d, is given by

$$V = IR \qquad \text{.. (14.20)}$$

where I is the instantaneous value of current. At every instant, the p.d. between points c and d must be equal to the p.d. across the source terminals a and b. Thus

$$IR = E_0 \sin \omega t$$

$$\text{or } I = \frac{E_0}{R} \sin \omega t = I_0 \sin \omega t \qquad \text{... (14.21)}$$

where I_0+E_0/R is the amplitude or peak value of the current. Graphs of functions for the instantaneous values, E and I, (Fig. 14.20), show that both the current and the voltage start at zero at the same time, reach maxima at the same time and have the same sinusoidal shape. The voltage and the current under these conditions are said to be in phase.

A voltmeter connected across the resistor would read the effective voltage whose value is $E_0/\sqrt{2}$. Similarly, an ammeter in the circuit would read $I_0/\sqrt{2}$ for the effective current. We,

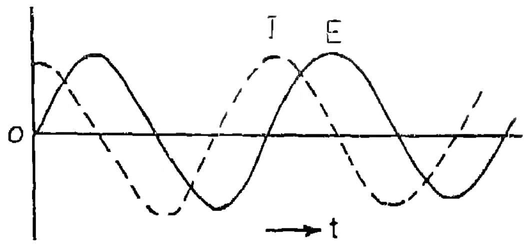


Fig. 14.20 Current is in phase with voltage in a resistive a.c. circuit

then, have

$$I_{eff} = \frac{I_0}{\sqrt{2}} = \frac{E_0}{R\sqrt{2}} = \frac{E_{eff}}{R} \qquad \text{... (14.22)}$$

14.10 A.C. Circuit Containing Inductance Only

Although it is difficult in practice to have a pure inductance, it is useful to consider the effects of a pure inductance in an a.c. circuit.

When a steady current flows through a pure inductance, the potential difference between points X and Y (Fig. 14.21) is zero as no resistance is offered to the current. If the current

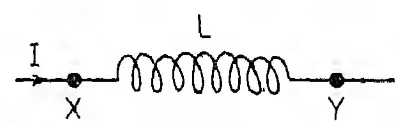


Fig. 14.21 P.D. across an inductor L is given by LdI/dt

is changing then an induced e.m.f. ($-L\,dI/dt$) exists in the inductor which opposes the change in the current. This makes the potential of Y different from X. The potential drop, when the current flows from X to Y is given by

$$V_X - V_Y = L\frac{dI}{dt} \qquad \text{... (14.23)}$$

Now consider the circuit of Fig. 14.22. Suppose an alternating current given by

$$I = I_0 \sin \omega t \qquad \text{... (14.24)}$$

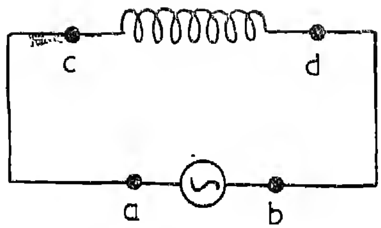


Fig. 14.22 A.C source with an inductance

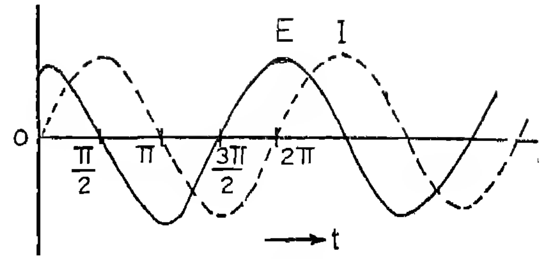


Fig. 14.23 Current I lags the voltage E by $\pi/2$ in an inductive circuit

flows in the inductor What must be the applied e.m.f. in the circuit in order to produce such a current ?

The instantaneous value of p.d. across L, between points c and d, is given by, equation (14 23),

$$V = L\frac{dI}{dt}$$

$$= L\frac{d}{dt}(I_0 \sin \omega t)$$

$$= L\omega I_0 \cos \omega t \qquad .. \quad (14.25)$$

Now, the p d between points c and d must, at every instant, be equal to the p d across the source terminals a and b. Hence the instantaneous value of applied e.m.f. is given by

$$E = L\omega I_0 \cos \omega t = E_0 \cos \omega t$$

$$= E_0 \sin(\omega t + \pi/2) \qquad . \quad (14.26)$$

where $E_0 = L\omega I_0$ is the amplitude or peak value of the applied e m f. Comparing with the resistive case, $E_0 = RI_0$, we find ωL plays the same role here as the resistance R in the earlier case, that is, it impedes the flow of current. The quantity

$$\omega L = X_L = 2\pi fL \qquad (14.27)$$

is called the inductive reactance of the circuit and has the unit of ohm

From equations (14 24) and (14 26), it is seen that the current and the voltage are 90° out of phase. Fig. 14.23 shows the way in which the instantaneous values, I and E, vary, I reaches maximum value a quarter of cycle later than E. One says that the pure inductance causes the current to lag behind the e.m f. in phase by $\pi/2$ (radians).

A voltmeter connected across the inductor would read $E_0/\sqrt{2}$ for the effective voltage and an ammeter in the circuit would read $I_0/\sqrt{2}$ for the effective current.

We then have,

$$I_{eff} = \frac{I_0}{\sqrt{2}} = \frac{E_0}{L\omega\sqrt{2}} = \frac{E_{eff}}{L\omega} = \frac{E_{eff}}{X} \qquad . \quad (14.28)$$

Inductive reactance X_L plays the same role here as the resistance R in equation (14.22).

Example 14 6

The current through a 1.0 henry inductor varies sinusoidally with an amplitude of 0.5 amperes and a frequency of 50 cycles per second. Calculate potential difference across the terminals of the inductor.

Solution

$I = I_0 \sin 2\pi ft$

p. d. across the inductor is given by,

$$V = L\frac{dI}{dt} = L\frac{d}{dt}(I_0 \sin 2\pi ft)$$

$$= 2\pi fL.I_0 \cos 2\pi ft$$

Amplitude of voltage is,

$$2\pi f L I_0 = 2\times3.14\times50\times1.0\times0.5 = 157 \text{ volts}$$

$$V = 157 \cos 100\pi t$$

Therefore,
The voltmeter will read the effective value

$$V_{eff} = \frac{V_0}{\sqrt{2}} = \frac{157}{\sqrt{2}} = 112 \text{ volts.}$$

EXAMPLE 14.7

What is the inductive reactance of a coil if the current through it is 80 mA and voltage across it is 40 V ?

Solution

These are the effective values hence,

$$X_L = \frac{V_{eff}}{I_{eff}} = \frac{40}{80\times10^{-3}} = 500 \text{ ohms}$$

EXAMPLE 14.8

At what frequency will a 0.5 henry inductor have a reactance of 2000 ohms ?

Solution

$$X_L = \omega L = 2\pi f L$$

$$2000 = 2\pi f\times0.5$$

$$f = \frac{2000}{\pi} = 637.$$

14.11. A. C Circuit Containing Capacitance Only

A capacitor consists of two plates of conducting material separated by an insulator. Its resistance is, therefore, practically, infinite. We might expect that when a capacitor is put into a circuit, d. c. or a. c. no current would flow. This is not so. Let us first consider a capacitor in a d.c. circuit.

Capacitor in a d.c. circuit : Consider the circuit shown in Fig. 14.24. As soon as we

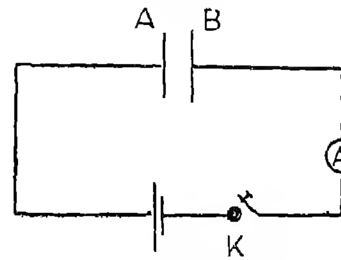


Fig. 14.24 Capacitor in a d.c circuit

press the key, electrons begin to flow from the negative terminal of the battery to the plate B and from plate A to the positive terminal of the battery. The plate A thus begins to acquire positive charge and the plate B, negative charge. This charging of the capacitor continues till the potential difference between the plates becomes equal to that across the battery terminals. Then, it ceases. This flow of charge is equivalent to a current. Thus a current does flow in the circuit, during the charging process, though not through the capacitor but in the remainder of the circuit The ammeter would, therefore, show a momentary deflection. The direction of current is from the plate B to the plate A via the battery. Its magnitude at any instant is given by the rate of growth of charge on the capacitor.

$$I = \frac{dQ}{dt} \qquad \text{... (14.29)}$$

The charging process can be extended in time if we include a resistance in the circuit, Fig. 14.25 It is found that the final charge on the capacitor, given by

$$Q_0 = V_0 C \qquad \text{... (14.30)}$$

where, V_0 = battery voltage, C = capicitance, is reached asymptotically as shown in Fig. 14.26. (Compare with the growth of current in a d.c. inductive circuit).

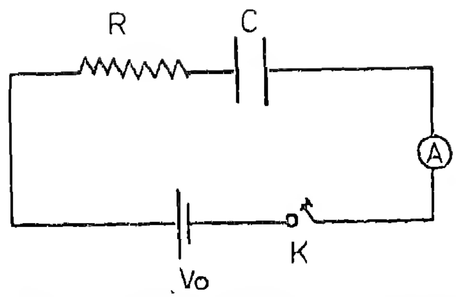


Fig. 14.25 Charging of a capacitor through a resistance

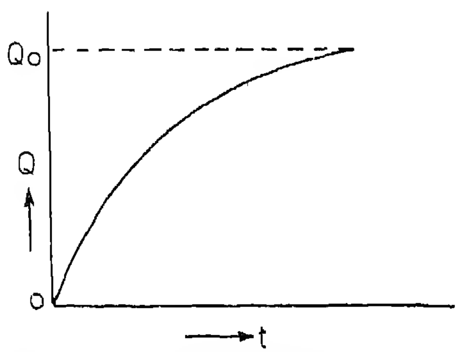


Fig. 14.26 Growth of charge on capacitor with time

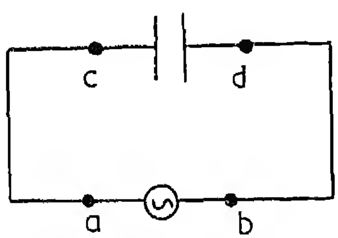


Fig. 14.27 A. C. source with a capacitor

Capacitor in an a.c. circuit: Now consider the capacitor in an a.c. circuit, Fig. 14.27. As the voltage from the source is continually changing, the charge on the capacitor is also continually changing. During a complete cycle, the capacitor is first charged in one direction, then discharged, again charged in the reverse direction and discharged. As this charging and discharging of the capacitor is taking place continuously, a continuous current exists in the circuit. Let us find out the nature of this current.

It is evident that the p.d. across the capacitor between points c and d, at every instant, has to be exactly the same as that across the source terminals a and b. Therefore the capacitor must charge and discharge in such a manner that the p.d. across it, V, is sinusoidal and equal to the applied e.m.f. at every instant. That is,

$$V = E = E_0 \sin \omega t$$

The charge on the capacitor, at any instant, is given by

$$Q = CV$$

Therefore, current at any instant is given by

$$\begin{aligned} I &= \frac{dQ}{dt} = C\frac{dV}{dt} = C\frac{d}{dt}(E_0 \sin \omega t) \\ &= E_0 C\omega \cos \omega t \\ &= I_0 \cos \omega t \\ &= I_0 \sin (\omega t + \pi/2) \qquad \dots (14.31) \end{aligned}$$

where,

$$I_0 = \frac{E_0}{1/C\omega} \qquad \dots (14.32)$$

Thus, in this case, the current is sinusoidal but 90° ahead of e m f., in phase. The wave form of E and I are shown in Fig. 14.28. The quantity

$$X_0 = \frac{1}{C\omega} = \frac{1}{2\pi fC} \qquad \dots (14.33)$$

is known as the capacitive reactance of the circuit. It plays the same role in this case, as the inductive reactance X_L in the inductive case. Its unit is ohm.

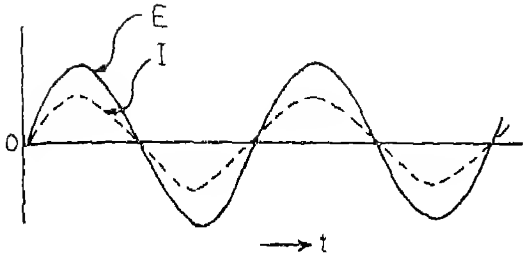


Fig. 14.28 Current I leads the voltage E by $\pi/2$ in a capacitive circuit

A voltmeter connected across the capacitor would read $E_0/\sqrt{2}$ for the effective voltage and an ammeter in the circuit would read $I_0/\sqrt{2}$ for the effective current. We, then, have

$$I_{eff}=\frac{I_0}{\sqrt{2}}=\frac{E_0}{\sqrt{2}\,(1/C\omega)}=\frac{E_{eff}}{1/C\omega}=\frac{E_{eff}}{X_C} \qquad \ldots (14.34)$$

EXAMPLE 14.9

What is the capacitive reactance of a 5-μf capacitor when it is part of a circuit whose frequency is (*i*) 50c/s (*ii*) 10^6c/s ?

Solution

(*i*) $X_C=\dfrac{1}{2\pi\times 50\times 5\times 10^{-6}}$ ohms

$= 637$ ohms

(*ii*) $X_C=\dfrac{1}{2\pi\times 10^6\times 5\times 10^{-6}}$

$=3.18\times 10^{-2}$ ohms.

14.12 LCR Circuit

A resistor, an inductor, or a capacitor, each of these circuit elements impedes an alternating current. The impeding effect is measured by the resistance (R), the inductive reactance (X_L) and the capacitive reactance (X_C), respectively, which are defined by

$$\frac{V}{I}=R \text{ (for resistor)}$$

$$\frac{V}{I}=X_L \text{ (for inductor)}$$

$$\frac{V}{I}=X_C \text{ (for capacitor)}$$

where V is the effective voltage across the circuit element and I the effective current through it.

Where a combination of these elements forms part of a circuit the total current impeding effect is defined, in an analogous manner by,

$$Z=\frac{V}{I} \qquad \ldots (14.35)$$

where Z is known as the impedance of the combination. Its unit, obviously, is ohm.

In Fig. 14.29 we have a series combination of a resistor of resistance R, an inductor of reactance X_L and a capacitor of reactance X_C. It turns out that the total impedance Z of the combination is given by

$$Z=\sqrt{R^2+(X_L-X_C)^2} \qquad \ldots (14.36)$$

and not by $Z=R+X_L+X_C$. The reason for this peculiar relationship is the fact that the reactances (X_L, X_C) arise because of a somewhat different physical mechanism than ordinary resistance (R). We cannot, therefore, combine them by simple addition. Equation (14.36) is a direct consequence of the vectorial addition of effective voltages in an a.c. circuit as discussed below.

In the circuit of Fig. 14.29, at any instant, the applied voltage is equal to the sum of the voltage drops across individual elements.

$$V=V_R+V_L+V_C \text{ (instantaneous values)}$$

However, if we measure the effective voltages

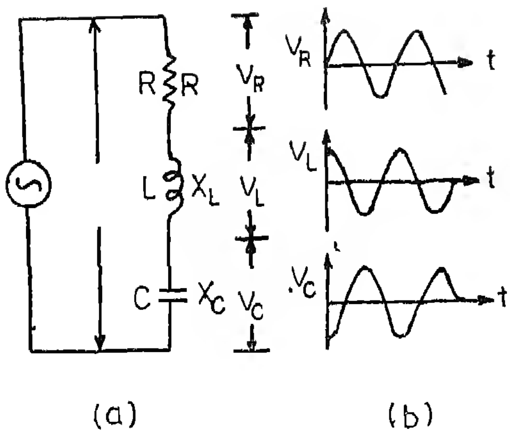


Fig 14.29 An LCR series circuit

(with a voltmeter) we would find that

$$V \neq V_R + V_L + V_C \text{ (effective values)}$$

The reason is, although same current passes through each component, the voltage across each of them bears a different phase relationship with the current and so the voltages are out of phase with one another. Mathematical analysis shows that effective voltages (and currents) add up vectorially. It is customary to treat them as vectors for the purpose of circuit analysis. Phase differences are represented by angles between the vectors.

Thus, in Fig. 14 30, the current I is represented by the vector OI; V_R, the voltage across R, being in phase with the current, is represented by vector OR; V_L, the voltage across L, being ahead of current by $\pi/2$, by vector OL and V_C, the voltage across C, since it lags behind the current by $\pi/2$, by vector OC. The resultant vector OP represents the effective voltage across the combination. Its magnitude is given by,

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} \quad \ldots (14.37)$$

Since $V_R = RI$, $V_L = X_L I$, $V_C = X_C I$ we have

$$V = \sqrt{R^2I^2 + (X_L - X_C)^2 I^2}$$

$$= \sqrt{R^2 + (X_L - X_C)^2}.I$$

or $$\frac{V}{I} = Z = \sqrt{R^2 + (X_L - X_C)^2} \quad \ldots (14.38)$$

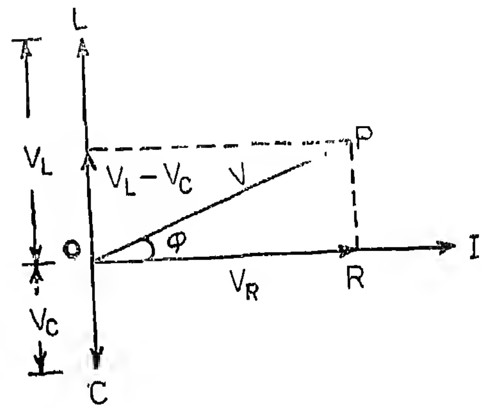


Fig. 14.30 Vectorial representation of voltages in an a.c. circuit

The relationship between impedance (Z), resistance (R) and reactances (X_L, X_C) is illustrated in Fig. 14.31 by constructing a vector diagram similar to Fig. 14.30.

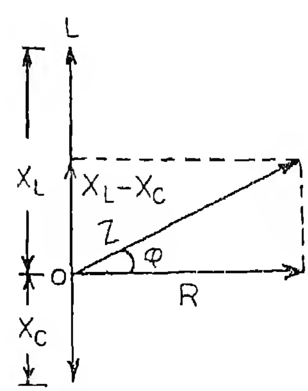


Fig. 14.31 Impedance diagram

It may be seen from Fig 14.30 that the current I lags behind the voltage V by phase angle ϕ Thus, the circuit is inductive. This is so because we have assumed $V_L > V_C$ (i e. $X_L > X_C$). If $X_L < X_C$, the combination would be capacitive and the current would lead the voltage. The phase angle, also called the angle of lead or lag, is given by

$$\tan\phi = \frac{X_L - X_C}{R} \text{ or } \cos\phi = \frac{R}{Z} \quad \ldots (14.39)$$

EXAMPLE 14.10

A resistor of 100 ohms, an inductor of 0.5 henry, and a capacitor of 10 microfarads are connected in series. A 220 volt 50 cycle alternating potential is connected across the group. Find (a) the impedance of the circuit, (b) the current, (c) the potential difference across each of the three elements, and (d) the phase angle between the current and the applied voltage. Construct the vector diagram for the voltages.

Solution

(*a*) $X_L = 2\pi fL = 2\pi \times 50 \times 0.5 = 157$ ohm.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 10 \times 10^{-6}}$$

$$= 318.4 \text{ ohm}$$

$$Z = \sqrt{(100)^2 + (157 - 318\ 4)^2}$$

$$= 189.5 \text{ ohm.}$$

(*b*) $I = \frac{220}{189.5} = 1.16$ ampere

(*c*) Voltage across the resistance $V_R = I \times R$ $= 1.16 \times 100 = 116$ volts

Voltage across the inductance V_L $= I \times X_L = 1.16 \times 157 = 182$ volts

Voltage across the capacitor $V_c =$ $I \times X_c = 1.16 \times 318.4 = 369$ volts

(*d*) $\cos\phi = \frac{R}{Z} = \frac{100}{189.5} = 0.5263$, $\phi = 58.3°$

Vector diagram for voltages is shown in Fig 14.32

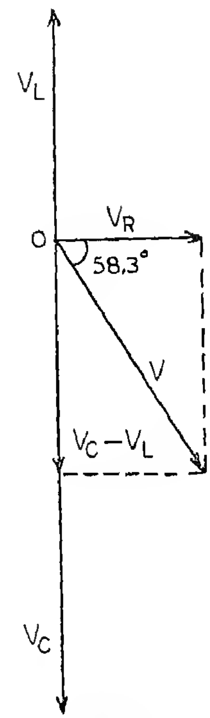


Fig. 14.32

Resonance

It may be seen if $X_L = X_C$, the impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

is at its minimum value and is simply equal to R. The circuit is purely resistive. The current is maximum. The applied voltage and the current are in phase. This is known as the condition of resonance and the frequency f_r at which this occurs is known as resonant frequency. This happens when

$$X_L = X_C$$

or $\quad 2\pi f_r L = \frac{1}{2 f_r \pi C}$

or $$f_r = \frac{1}{2\pi\sqrt{LC}} \quad ...(14.40)$$

14.13 Transformer

One of the most useful applications of electromagnetic induction is the transformer. The construction of a simple transformer is shown in Fig. 14 33. Two coils, each consisting of many turns of wire are wound on a

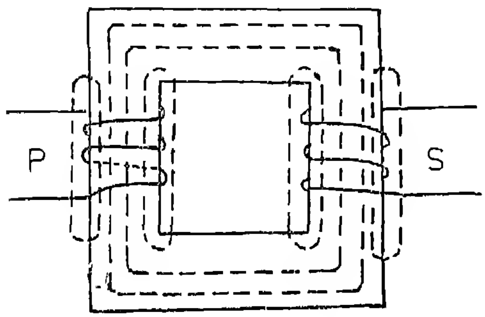


Fig. 14.33 Transformer : P-Primary, S-Secondary

continuous iron core. One of these coils called the 'primary' is connected to an a.c. source. The other coil, called the 'secondary' is connected to the 'load' which may be a resistance or any other electrical device to which electric power is to be supplied.

The alternating current in the primary produces an alternating magnetic flux in the core which passes through the secondary coil also. As there is very little 'leakage' the magnetic flux through the secondary is almost the same as through the primary. This changing magnetic flux produces an induced e m.f. in the secondary and it also causes a self-induced back e.m f. in the primary.

Consider the situation when no load is attached to the secondary, that is, its terminals are open. Let N_1 and N_2 be the number of turns in the primary and secondary, respectively. The induced e.m.f. in the primary is given by

$$E_1 = -N_1\frac{d\phi}{dt} \quad ... (14.41)$$

where, I_1 is the current in the primary and ϕ the magnetic flux in the core, at any instant. The induced e.m.f. in the secondary is given by

$$E_2 = -N_2\frac{d\phi}{dt} \quad .. (14.42)$$

Thus,

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} \quad ... (14.43)$$

The value of induced e.m.f. in the primary can be shown to be almost equal to the applied e.m.f.

Let the applied e.m.f. to the primary be $E = E_0 \sin \omega t$. In the primary circuit, the self induced e.m.f. E_1 acts opposite to the applied e m.f. (Lenz's law). At any instant it is the difference, $E—E_1$, which sends the current I_1 through the resistance R of the primary,

Thus,

$$E - E_1 = RI_1$$

However, R is very small, and so the term RI_1 can be neglected. Hence

$$E = E_1 \quad .. (14.44)$$

Thus, in equation (14.43) E_1 may be described as the input e.m f. given to the primary and E_2 as the out-put e.m.f from the secondary. We have

$$\frac{E_2}{E_1} = \frac{\text{output e.m.f}}{\text{input e m f.}} = \frac{N_2}{N_1} \quad ... (14.45)$$

If V_1 and V_2 be the effective values of e.m.f s, then

$$\frac{V_2}{V_1}=\frac{N_1}{N_2} \quad \text{.. (14.46)}$$

The quantity N_2/N_1 is called the 'turns ratio' of the transformer. When $N_2>N_1$, $V_2>V_1$ the transformer is known as stop-up transformer. If $N_2<N_1$, $V_2<V_1$ the transformer is known as step-down transformer.

The law of conservation of energy requires that the energy delivered to the secondary circuit must be equal to or less than the energy supplied to the primary. Assuming the transformer to be ideal, with no energy losses, the average power input must be equal to the average power output. Therefore,

$$V_1I_1 = V_2I_2 \quad \text{.. (14.47)}$$

where the voltages and currents are the effective values. From eq. (14.47) we have

$$\frac{I_2}{I_1}=\frac{V_1}{V_2}=\frac{N_1}{N_2} \quad \text{... (14.48)}$$

The current in the secondary decreases in the same ratio as that by which the voltage increases.

The efficiency of a transformer is defined as

$$n=\frac{\text{Power output}}{\text{Power input}}$$

In real transformers, the efficiency is fairly high (90-99 per cent) though not 100 per cent. There are several causes for power loss. The main losses are two : (i) the I^2R loss-due to heating of copper wires used in the windings. This can be minimized by using thick wires. (ii) Core loss-due to work done in carrying the iron core through cycles of magnetization and demagnetization. This is minimized by the choice of iron with special magnetic properties. In addition, there is power loss due to eddy currents which is reduced by using laminated iron core (Fig. 14.34). There is also some loss due to flux leakage but it is quite small.

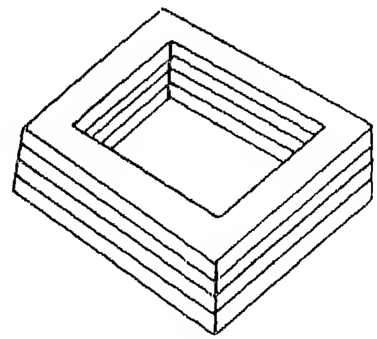


Fig. 14.34 Laminated core

Transmission of electric power

Electric power stations are, generally, situated in remote areas where it is cheaper to produce electric power. This power has to be transmitted to the cities and areas where it is needed. This is done by transmission lines which consist of two parallel wires for carrying current from and to the power station.

To avoid the loss of power due to I^2R losses in the line wire, the output, voltage of the generator is first 'transformed' to a much higher value by a step-up transformer. It converts the electric power at low voltage and high current to the same power at higher voltage and lower current. Due to reduction in the value of current, the I^2R losses in the lines are reduced. To appreciate the economy in transmitting power at high voltage, let us consider the following example

Suppose a power generator produces 25 KW of power at 100 amperes and 250 volts. It is desired to deliver this power to a consumer 1 Km away on a transmission line whose resistance is, say, 1 ohm. The line loss is given by $I^2R = 100^2\times1 = 10{,}000$ W $= 10$ KW. Thus

40 per cent of power would be wasted. If we step-up the voltage by a transformer to 2500 volts, the current in the lines would reduce to 10 amperes (Fig 14.35). The I^2R loss would come to $10^2 \times 1 = 100W$ which is negligible.

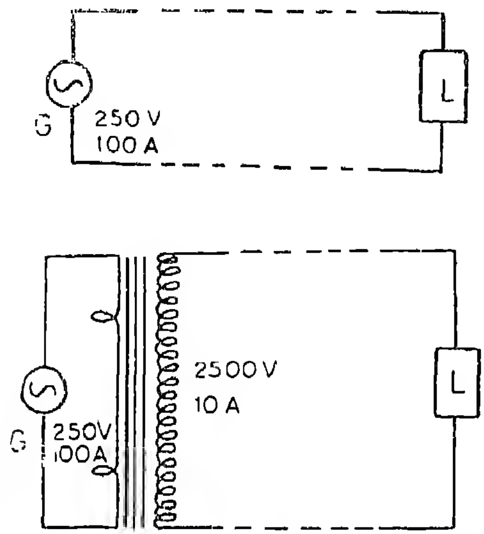


Fig. 14.35 Transmission of power at higher voltage reduces line losses. G - Generator, T - Step-up transformer, L - Load.

A typical power generator gives an output of 1000 KW at 6600 volts In practice this voltage is stepped up to 132000 volts before transmission. The cables used for transmitting power over long distances are suspended by large porcelain insulators from large steel structures (pylons) The main transmission lines from power stations form part of a common system called the 'grid' which covers a large region of the country. Power from all the power stations in the region is fed into the grid which forms a common pool from which power can be drawn where needed. This allows an efficient power distribution and acts as a safeguard for ensuring a minimum power supply to consumers in the event of failure of power generation at some station. From the grid, the power is fed to the cities at 33000 V, the stepping down is done outside the city. Then again at a sub-station, the supply is stepped down to 6600 volt. Power is supplied to the big consumers like factories at this voltage which they can further step-down according to their needs. For ordinary domestic consumers the voltage is again reduced to 220 V.

Exercises

14.1 What are the dimensions of magnetic flux ?

14.2 Mark the current direction in the secondary windings of Fig 14.36 as the switch is closed.

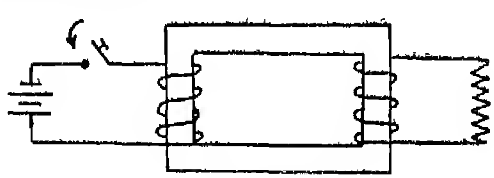


Fig. 14.36

large and may cause a spark to jump across the partially open switch. When the initial current is large and the inductance high, the spark may be sufficiently strong to cause damage to switch contacts and insulation. Switches for d.c. circuits, in such cases, are so designed as to allow the current to decay gradually and thus prevent sparking.

14.7 Some Phenomena Connected with Inductance

(i) *Eddy currents*

When a sheet of metal is placed in a changing magnetic field, induced currents are set up in the sheet which oppose these changes. These are known as eddy currents. The currents are circular, their sense of flow can be ascertained by applying Lenz's law. Fig 14.17 shows some eddy currents in a metal sheet placed in an increasing magnetic field pointing in to the plane of paper.

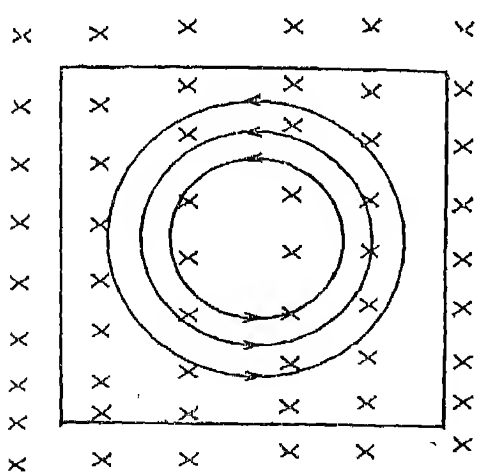


Fig. 14.17 Eddy currents

Since the resistance of metals is quite low, these currents may be quite large and may produce considerable heating effects. Eddy currents are used in heating small metal specimens by putting them in rapidly changing magnetic fields produced by high frequency alternating currents. It is called inducation heating.

Eddy currents are considered undesirable in electrical appliances and machinery, where iron is widely used, as they cause unnecessary heating and wastage of power. This is avoided by building up the desired iron part with thin sheets of the metal which are insulated from each other. The planes of sheets are placed perpendicular to the direction of currents that would be set up by the e.m.f induced in the material. The insulation between the sheets then offers high resistance to the induced e.m.f. and the eddy currents are substantially reduced.

(ii) *Electromagnetic damping*

When a current is passed through a galvanometer the galvanometer coil usually suffers a few to and fro oscillations before settling down to its proper deflected position. The motion of the coil is damped largely because of electromagnetic damping—as the coil moves in the magnetic field, a counter e.m.f. is induced in the coil which opposes its motion. The electromagnetic damping can be further increased by winding the coil on a metallic frame. As the frame moves, eddy currents are generated in the frame which use up energy and hence damp the motion. In a properly constructed galvanometer oscillations can be prevented completely—the coil deflects and stays at its final position.

14.8 Alternating Current

Let a source of alternating e.m.f. (represented by the symbol (~) be connected across

a resistance R, Fig. 14.18. As the e.m.f., $E = E_0 \sin \omega t$, changes with time, the current

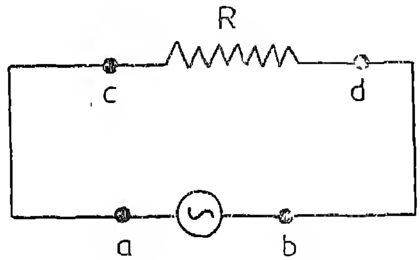


Fig. 14.18 A. C. source with resistance

through the circuit also changes. At any instant, the current would be given by Ohm's law

$$I = \frac{E}{R} = \frac{E_0}{R} \sin \omega t$$

$$= I_0 \sin \omega t = I_0 \sin 2\pi ft. \qquad \dots (14.16)$$

where $I_0 = E_0/R$ is the maximum or peak value of current, f is the frequency and t is the time. The current is sinusoidal and is known as alternating current. Its variation with time

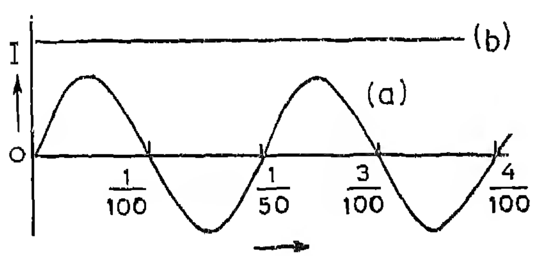


Fig. 14.19 (a) Wave form of alternating current
(b) steady current

is shown in Fig. 14.19, the time scale chosen is for the commonly used 50 c/s alternating current. For half the cycle the current flows in one direction and for the other half in the opposite direction. A steady direct current is also shown in the same diagram for comparison.

Since an a.c. current[*] varies in magnitude continuously and changes direction periodically, the effects produced by it (such as the magnetic effect) will also vary with time. The question immediately arises as to how can we measure such a current? How do we define an ampere of an alternating current? There is only one effect which is independent of the direction of the electric current—the heating effect, which we make use of for this purpose. We compare the heating effect of an alternating current with that of a direct current and define the former in terms of the latter.

The effective value of an alternating current, I_{eff} is, therefore, defined as that magnitude of direct current which produces the same heating effect in a given resistance as the given alternating current. Thus an a.c. ampere would be an a.c. current that produces a heating effect equal to that of one d.c. ampere.

It is found that an a.c. current $I = I_0 \sin \omega t$, produces the same heating effect as d.c. current of $I_0/\sqrt{2}$. The value

$$I_{eff} = \frac{I_0}{\sqrt{2}} = 0.707\, I_0 \qquad \dots (14.17)$$

is known as the effective value of the a.c. current. This is also known, sometimes, as root-mean-square or r.m.s. current.[†]

The effective or r.m.s. value of an a.c. voltage is defined in an exactly similar way and we have

$$E_{eff} = \frac{E_0}{\sqrt{2}} = 0.707\, E_0 \qquad \dots (14.18)$$

* Since a.c. means alternating current, it may seem wrong to use the terms, an a.c. current or an a.c. voltage, from language point of view, but it has now become a common practice to use these terms.

† The reason is, the square root of the mean of the square of the current i.e. $\sqrt{(I^2)}^{\frac{1}{2}}$ av $= I_0/\sqrt{2}$

Hereafter, whenever numerical values of a.c. voltage and current are given, it is effective values which are meant unless specifically stated otherwise. Thus an a.c. current of 10A means that the current varies in a sinusoidal manner with a peak value of $10\sqrt{2}$ A. An electric supply at 220v means the peak value of voltage is $220\sqrt{2} = 310$v.

14.9 A. C. Circuit Containing Resistance Only

Figure 14.18 shows a resistance R connected in series with an a.c. source of e.m.f.

$$E = E_0 \sin \omega t \qquad .. (14.19)$$

At any instant, the potential difference across the resistance R, between the points c and d, is given by

$$V = IR \qquad . (14.20)$$

where I is the instantaneous value of current. At every instant, the p.d. between points c and d must be equal to the p.d. across the source terminals a and b. Thus

$$IR = E_0 \sin \omega t$$

$$\text{or } I = \frac{E_0}{R} \sin \omega t = I_0 \sin \omega t \qquad .. (14.21)$$

where $I_0 + E_0/R$ is the amplitude or peak value of the current. Graphs of functions for the instantaneous values, E and I, (Fig. 14.20), show that both the current and the voltage start at zero at the same time, reach maxima at the same time and have the same sinusoidal shape. The voltage and the current under these conditions are said to be in phase.

A voltmeter connected across the resistor would read the effective voltage whose value is $E_0/\sqrt{2}$. Similarly, an ammeter in the circuit would read $I_0/\sqrt{2}$ for the effective current. We,

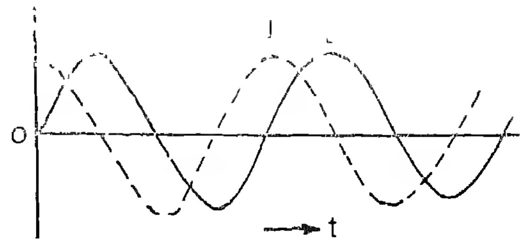


Fig. 14.20 Current is in phase with voltage in a resistive a.c. circuit

then, have

$$I_{eff} = \frac{I_0}{\sqrt{2}} = \frac{E_0}{R\sqrt{2}} = \frac{E_{eff}}{R} \qquad .. (14.22)$$

14.10 A.C. Circuit Containing Inductance Only

Although it is difficult in practice to have a pure inductance, it is useful to consider the effects of a pure inductance in an a.c. circuit.

When a steady current flows through a pure inductance, the potential difference between points X and Y (Fig 14.21) is zero as no resistance is offered to the current. If the current

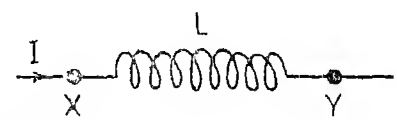


Fig. 14.21 P.D. across an inductor L is given by LdI/dt

is changing then an induced e.m.f. (—LdI/dt) exists in the inductor which opposes the change in the current. This makes the potential of Y different from X. The potential drop, when the current flows from X to Y is given by

$$V_X - V_Y = L\frac{dI}{dt} \qquad ... (14.23)$$

Now consider the circuit of Fig. 14.22. Suppose an alternating current given by

$$I = I_0 \sin \omega t \qquad ... (14.24)$$

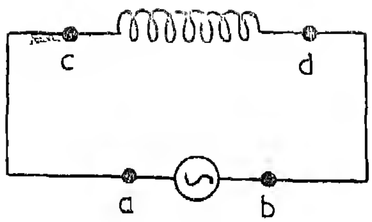


Fig. 14.22 A.C source with an inductance

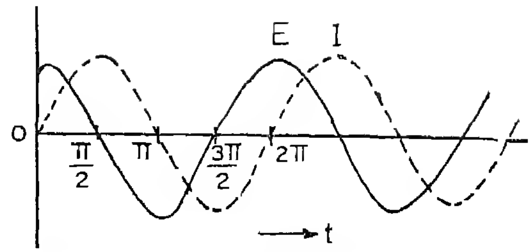


Fig. 14.23 Current I lags the voltage E by $\pi/2$ in an inductive circuit

flows in the inductor What must be the applied e.m f. in the circuit in order to produce such a current ?

The instantaneous value of p d. across L, between points c and d, is given by, equation (14.23),

$$V = L\frac{dI}{dt}$$

$$= L\frac{d}{dt}(I_0 \sin \omega t)$$

$$= L\omega I_0 \cos \omega t \qquad (14.25)$$

Now, the p d between points c and d must, at every instant, be equal to the p d. across the source terminals a and b. Hence the instantaneous value of applied e m f. is given by

$$E = L\omega I_0 \cos \omega t = E_0 \cos \omega t$$

$$= E_0 \sin (\omega t+\pi/2) \qquad \ldots (14.26)$$

where $E_0 = L\omega I_0$ is the amplitude or peak value of the applied e.m f. Comparing with the resistive case, $E_0 = RI_0$, we find ωL plays the same role here as the resistance R in the earlier case, that is, it impedes the flow of current. The quantity

$$\omega L = X_L = 2\pi fL \qquad (14.27)$$

is called the inductive reactance of the circuit and has the unit of ohm.

From equations (14.24) and (14.26), it is seen that the current and the voltage are 90° out of phase. Fig. 14.23 shows the way in which the instantaneous values, I and E, vary, I reaches maximum value a quarter of cycle later than E. One says that the pure inductance causes the current to lag behind the e.m f. in phase by $\pi/2$ (radians).

A voltmeter connected across the inductor would read $E_0/\sqrt{2}$ for the effective voltage and an ammeter in the circuit would read $I_0/\sqrt{2}$ for the effective current.

We then have,

$$I_{eff} = \frac{I_0}{\sqrt{2}} = \frac{E_0}{L\omega\sqrt{2}} = \frac{E_{eff}}{L\omega} = \frac{E_{eff}}{X} \qquad \ldots (14.28)$$

Inductive reactance X_L plays the same role here as the resistance R in equation (14.22).

EXAMPLE 14.6

The current through a 1.0 henry inductor varies sinusoidally with an amplitude of 0.5 amperes and a frequency of 50 cycles per second. Calculate potential difference across the terminals of the inductor.

Solution

$I = I_0 \sin 2\pi ft$

p. d. across the inductor is given by,

$$V = L\frac{dI}{dt} = L\frac{d}{dt}(I_0 \sin 2\pi ft)$$

$$= 2\pi fL.I_0 \cos 2\pi ft$$

$$E = 100\pi \times 1000 \times 0.10 \times 0.05 \times 100 \times 10^{-4} \sin 100\,\pi t$$
$$= 5\pi \sin 100\,\pi t$$

Maximum value of induced e.m.f.,
$$E_0 = 5\pi = 15.7 \text{ volts}$$

When $\omega t = 45°$, $E = E_0 \sin 45°$
$$= 15.7 \times \sqrt{\tfrac{1}{2}} = 11.2 \text{ volts.}$$

14.4 The Generator or Dynamo

What we have described above is the principle of a generator, a means of producing electric current continually by mechanical means. It is the most important method of converting mechanical energy into electrical energy

Fig. 14.8 shows schematically the essential parts of a generator. A coil abcd capable of

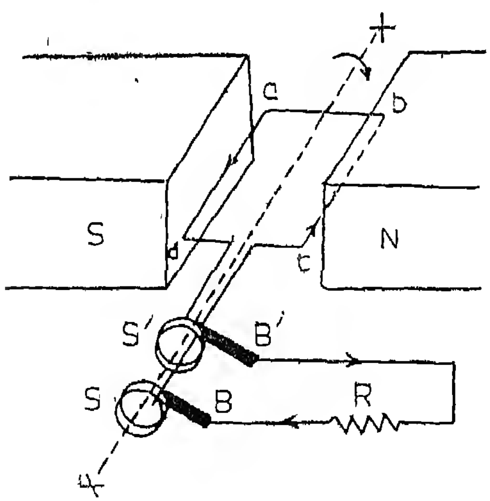


Fig. 14.8 A C. Generator

rotation about the axis XX is situated in the field of a magnet NS. As the coil rotates an alternating e.m.f. develops in the coil which is fed to the external circuit by means of a pair of metal slip rings, S and S′ which are fixed rigidly to the same shaft which is used to rotate the coil Each ring is connected to one terminal of the coil and rotates with the coil The rings maintain sliding contact with the brushes, B and B′. The output e m.f. of the generator is alternating as shown in Fig 14 7. As the coil rotates, the current flows out through the brush B for one half of a revolution and through the brush B′ for the next half The current through the external resistance R is also alternating

If one directional, *i.e* direct current is desired, the slip rings are replaced by what is known as a split ring commutator as shown in Fig. 14.9 The commutator rotates with the coil. As before brushes make sliding contact

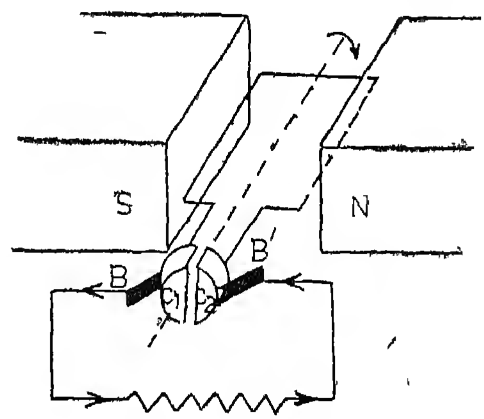


Fig. 14.9 D. C. Generator

with the commutator for half the rotation when C_1 is positive, it is in contact with the brush B, for the other half of rotation it is C_2 which is positive and is in contact with B. Thus the current always leaves the generator through the brush B and we get a direct current in the external circuit. The output, however, is pulsating as shown in Fig 14.10. If we have several coils, uniformally spaced, as in Fig. 14.11, all

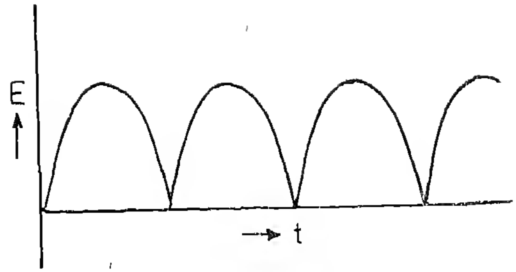


Fig. 14.10 Output e.m.f. from a single coil D C Generator

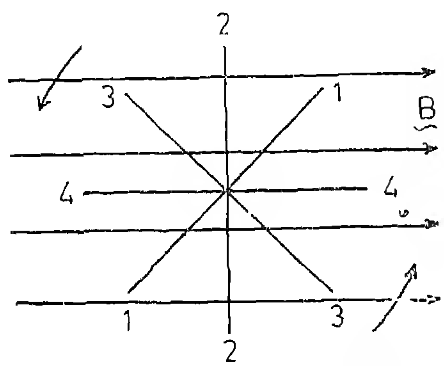


Fig. 14.11 Four coils rotating in a magnetic field

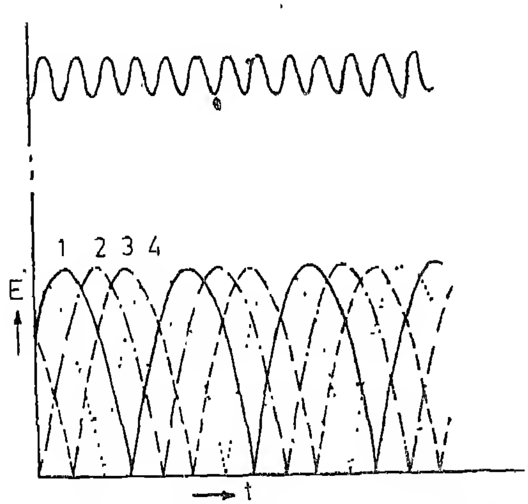


Fig. 14.12 Output e.m.f. from four coils rotating in a magnetic field

connected in series, the maximum value of e m f. occurs in each coil at different instants and the net effect is an almost constant unidirectional e m.f. with a small ripple. Fig 14 12 shows results achieved with four coils More coils would give an even more smooth output.

14.5 Mutual Inductance

Consider two coils P and S placed near each other, Fig. 14.13(a). If coil P carries a current, it produces a magnetic field which produces a magnetic flux through S If the current in the coil P, called the primary, is changed, it would cause an induced e m f. in the coil S, called the secondary.

Let the current through primary coil at any instant be I_1. Then the magnetic flux at any part of the secondary coil will be proportional to I_1 that is

$$\phi_2 \propto I_1$$

Therefore, the induced e.m.f. in the secondary, when I_1 changes, is given by

$$E = -\frac{d\phi_2}{dt}$$

$$\text{i e.} \quad E \propto -\frac{dI_1}{dt} = -\frac{MdI_1}{dt} \qquad \text{... (14.14)}$$

where M is the constant of proportionality, called the mutual inductance of the two coils. It is defined as the e m f induced in the secondary by a unit rate of change of current in the primary. When E is expressed in volts, and $\frac{dI_1}{dt}$ in amperes/second, M is in volt/second/ampere, called henry (H). The mutual inductance between the circuits is 1H, if a rate of change of current of 1A/s in the primary produced an induced e.m.f. of 1 volt in the secondary.

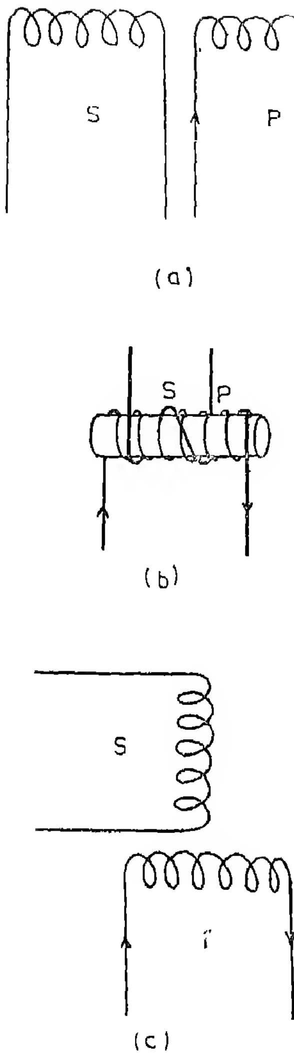


Fig 14.13 (a) Mutual induction Variation of current in primary coil P induces an e.m.f in the secondary coil S
(b) case of largest value of M
(c) case of least value of M.

M depends upon the number of turns in the coils, their geometrical shape and their separation. It is maximum when the entire flux of the primary links with the secondary. Thus the value of M will be minimum when one coil is perpendicular to another as in Fig. 14.13(c) The mutual inductance is further increased, if the coils in Fig 14 13(b) are wound over an iron core, by a factor μ, where μ is the permeability of iron.

14.6 Self-inductance

When a current flows in a coil, it gives rise to a magnetic flux through the coil itself. If the current strength changes, the flux changes and an e.m f is induced in the coil. This e.m f. is called self-induced e.m f. and the phenomenon is known as self-induction.

It is easy to see that the flux through the coil is proportional to the current through it, *i e.*

$$\phi \propto I$$

and, therefore, the induced e.m.f E is given by

$$E = \frac{-d\phi}{dt}$$

$$i.e. \quad E \propto -\frac{dI}{dt} = -L\frac{dI}{dt} \qquad \ldots (14.15)$$

where L is called the self-inductance of the coil. Like M, L is also measured in henry. A coil has self-inductance of 1 henry if an e m f of 1 volt is produced in it when the current passing through it changes at the rate of one ampere per second.

Self-inductance, often called inductance, is a constant of the coil It depends upon the number of turns, area of cross section, and the permeability of the core material. The larger the number of turns and area of cross section of a coil, the larger is its inductance. If coil is wound over an iron core, the self-inductance

increases by a factor μ where μ is the permeability of iron A coil possessing an appreciable inductance is known as an inductor

EXAMPLE 14.5

What e.m.f will be induced in a 10H inductor in which the current changes from 10 amperes to 7 amperes in 9.0×10^{-2} seconds ?

Solution

$$E = -L\frac{dI}{dt}$$

$$= -L\frac{(I_2 - I_1)}{t}$$

$$= -10\frac{(7-10)}{9.0 \times 10^{-2}}$$

$$= 333 \text{ volts}$$

Inductance in a d c circuit .

Figure 14 14 shows an inductance L and a resistance R (including the resistance of the

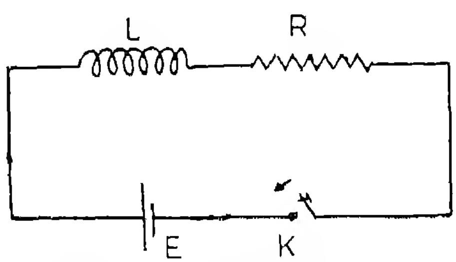


Fig. 14.14 Inductance in a d.c circuit

coil L) connected to a battery of e.m.f. E. When the key K is closed, the current begins to grow. As the current increases, the magnetic field associated with it also increases and so does the magnetic flux through the coil. This increasing flux induces an e.m.f which produces a current opposed to the one which is growing, (Lenz's law). This limits the rate of rise of the current and it takes some time before the current reaches a steady value given by Ohm's law, $I_0 = E/R$ The effect of inductance, therefore, is to increase the time taken by the current to reach its limiting value I_0. The length of this time depends on the value of L and may vary from a few milliseconds to several seconds. The manner in which I attains the value I_0 is shown in Fig. 14.15 for two values of L.

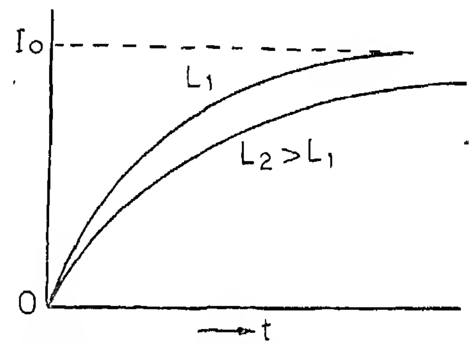


Fig. 14.15 Growth of current in an inductive circuit

Similarly, when a current flowing in a circuit containing inductance is interrupted, an induced e.m.f. is set up in the circuit which tends to maintain the current. Thus in Fig. 14.16 if the switch K is opened, the current decreases rapidly, if this change takes place very rapidly, the induced e.m.f. may be quite

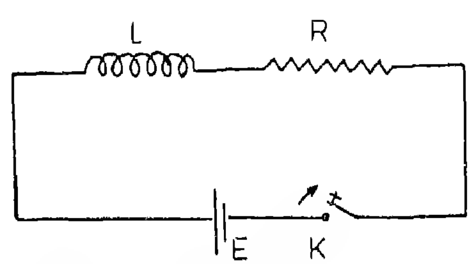


Fig. 14.16 Opening the switch in an inductive circuit leads to sparking

It may be seen from Fig. 14 30 that the current I lags behind the voltage V by phase angle ϕ Thus, the circuit is inductive. This is so because we have assumed $V_L > V_C$ (*i e.* $X_L > X_C$). If $X_L < X_C$, the combination would be capacitive and the current would lead the voltage. The phase angle, also called the angle of lead or lag, is given by

$$\tan\phi = \frac{X_L - X_C}{R} \text{ or } \cos\phi = \frac{R}{Z} \quad \dots (14.39)$$

EXAMPLE 14.10

A resistor of 100 ohms, an inductor of 0 5 henry, and a capacitor of 10 microfarads are connected in series. A 220 volt 50 cycle alternating potential is connected across the group. Find (a) the impedance of the circuit, (b) the current, (c) the potential difference across each of the three elements, and (d) the phase angle between the current and the applied voltage. Construct the vector diagram for the voltages.

Solution

(*a*) $X_L = 2\pi fL = 2\pi \times 50 \times 0.5 = 157$ ohm.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 10 \times 10^{-6}}$$

$$= 318\ 4 \text{ ohm}$$

$$Z = \sqrt{(100)^2 + (157 - 318\ 4)^2}$$

$$= 189.5 \text{ ohm.}$$

(*b*) $I = \dfrac{220}{189.5} = 1.16$ ampere

(*c*) Voltage across the resistance $V_R = I \times R = 1.16 \times 100 = 116$ volts

Voltage across the inductance $V_L = I \times X_L = 1.16 \times 157 = 182$ volts

Voltage across the capacitor $V_C = I \times X_C = 1.16 \times 318\ 4 = 369$ volts

(*d*) $\cos\phi = \dfrac{R}{Z} = \dfrac{100}{189.5} = 0.5263$, $\phi = 58.3°$

Vector diagram for voltages is shown in Fig. 14 32

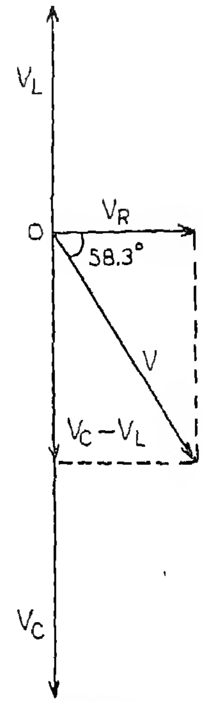


Fig. 14.32

Resonance

It may be seen if $X_L = X_c$, the impedance

$$Z = \sqrt{R^2 + (X_L - X_c)^2}$$

its minimum value and is simply equal to the circuit is purely resistive. The current ...mum. The applied voltage and the ... are in phase. This is known as the ...n of resonance and the frequency f_r at ...his occurs is known as resonant fre- ... This happens when

$$= X_c$$

$$2\pi f_r L = \frac{1}{2f_r\pi C}$$

or $$f_r = \frac{1}{2\pi\sqrt{LC}} \quad \ldots(14.40)$$

14 13 Transformer

One of the most useful applications of electromagnetic induction is the transformer The construction of a simple transformer is shown in Fig 14.33. Two coils, each consisting of many turns of wire are wound on a

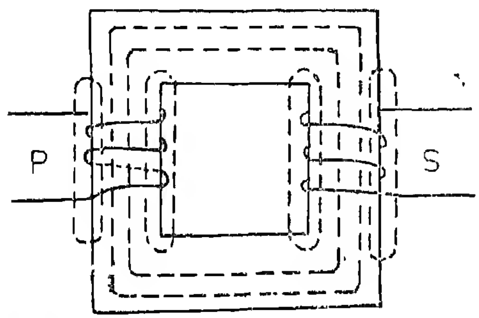


Fig. 14.33 Transformer · P-Primary, S-Secondary

continuous iron core. One of these coils called the 'primary' is connected to an a.c. source. The other coil, called the 'secondary' is connected to the 'load' which may be a resistance or any other electrical device to which electric power is to be supplied.

The alternating current in the primary produces an alternating magnetic flux in the core which passes through the secondary coil also. As there is very little 'leakage' the magnetic flux through the secondary is almost the same as through the primary. This changing magnetic flux produces an induced e m f. in the secondary and it also causes a self-induced back e m.f in the primary.

Consider the situation when no load is attached to the secondary, that is, its terminals are open. Let N_1 and N_2 be the number of turns in the primary and secondary, respectively The induced e.m f. in the primary is given by

$$E_1 = -N_1\frac{d\phi}{dt} \quad \ldots(14\ 41)$$

where, I_1 is the current in the primary and ϕ the magnetic flux in the core, at any instant. The induced e.m f in the secondary is given by

$$E_2 = -N_2\frac{d\phi}{dt} \quad (14.42)$$

Thus,

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} \quad \ldots(14.43)$$

The value of induced e m f in the primary can be shown to be almost equal to the applied e m f

Let the applied e.m f. to the primary be $E = E_0 \sin \omega t$. In the primary circuit, the self induced e m f. E_1 acts opposite to the applied e m f. (Lenz's law). At any instant it is the difference, $E—E_1$, which sends the current I_1 through the resistance R of the primary,

Thus,

$$E - E_1 = RI_1$$

However, R is very small, and so the term RI_1 can be neglected. Hence

$$E = E_1 \quad .(14.44)$$

Thus, in equation (14.43) E_1 may be described as the input e.m f. given to the primary and E_2 as the out-put e.m.f from the secondary. We have,

$$\frac{E_2}{E_1} = \frac{\text{output e.m f.}}{\text{input e m f.}} = \frac{N_2}{N_1} \quad ..(14.45)$$

If V_1 and V_2 be the effective values of e m f s, then

$$\frac{V_2}{V_1} = \frac{N_1}{N_2} \quad \dots (14.46)$$

The quantity N_2/N_1 is called the 'turns ratio' of the transformer When $N_2 > N_1$, $V_2 > V_1$ the transformer is known as stop-up transformer. If $N_2 < N_1$, $V_2 < V_1$ the transformer is known as step-down transformer.

The law of conservation of energy requires that the energy delivered to the secondary circuit must be equal to or less than the energy supplied to the primary. Assuming the transformer to be ideal, with no energy losses, the average power input must be equal to the average power output. Therefore,

$$V_1 I_1 = V_2 I_2 \quad \dots (14.47)$$

where the voltages and currents are the effective values. From eq (14.47) we have

$$\frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{N_1}{N_2} \quad \dots (14.48)$$

The current in the secondary decreases in the same ratio as that by which the voltage increases.

The efficiency of a transformer is defined as

$$n = \frac{\text{Power output}}{\text{Power input}}$$

In real transformers, the efficiency is fairly high (90-99 per cent) though not 100 per cent. There are several causes for power loss The main losses are two : (i) the I^2R loss-due to heating of copper wires used in the windings. This can be minimized by using thick wires. (ii) Core loss-due to work done in carrying the iron core through cycles of magnetization and demagnetization. This is minimized by the choice of iron with special magnetic properties. In addition, there is power loss due to eddy currents which is reduced by using laminated iron core (Fig 14 34). There is also some loss due to flux leakage but it is quite small.

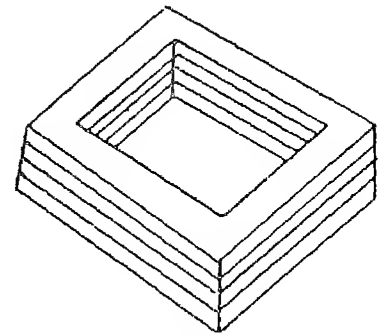


Fig 14.34 Laminated core

Transmission of electric power

Electric power stations are, generally, situated in remote areas where it is cheaper to produce electric power This power has to be transmitted to the cities and areas where it is needed. This is done by transmission lines which consist of two parallel wires for carrying current from and to the power station.

To avoid the loss of power due to I^2R losses in the line wire, the output, voltage of the generator is first 'transformed' to a much higher value by a step-up transformer. It converts the electric power at low voltage and high current to the same power at higher voltage and lower current Due to reduction in the value of current, the I^2R losses in the lines are reduced To appreciate the economy in transmitting power at high voltage, let us consider the following example

Suppose a power generator produces 25 KW of power at 100 amperes and 250 volts It is desired to deliver this power to a consumer 1 Km away on a transmission line whose resistance is, say, 1 ohm. The line loss is given by $I^2R = 100^2 \times 1 = 10{,}000$ W = 10 KW. Thus

40 per cent of power would be wasted. If we step-up the voltage by a transformer to 2500 volts, the current in the lines would reduce to 10 amperes (Fig. 14.35). The I^2R loss would come to $10^2 \times 1 = 100W$ which is negligible.

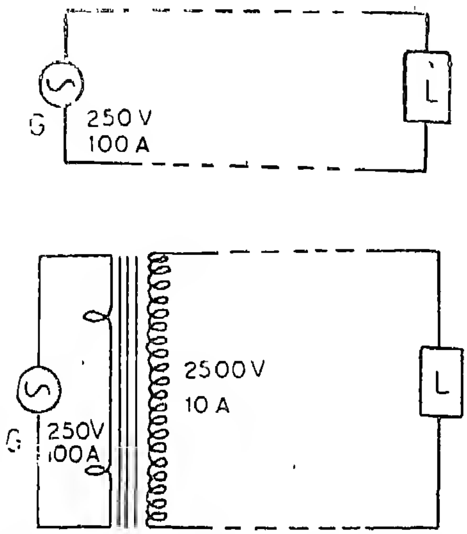


Fig. 14.35 Transmission of power at higher voltage reduces line losses. G - Generator, T - Step-up transformer, L - Load.

A typical power generator gives an output of 1000 KW at 6600 volts. In practice this voltage is stepped up to 132000 volts before transmission. The cables used for transmitting power over long distances are suspended by large porcelain insulators from large steel structures (pylons). The main transmission lines from power stations form part of a common system called the 'grid' which covers a large region of the country. Power from all the power stations in the region is fed into the grid which forms a common pool from which power can be drawn where needed. This allows an efficient power distribution and acts as a safeguard for ensuring a minimum power supply to consumers in the event of failure of power generation at some station. From the grid, the power is fed to the cities at 33000 V, the stepping down is done outside the city. Then again at a sub-station, the supply is stepped down to 6600 volt. Power is supplied to the big consumers like factories at this voltage which they can further step-down according to their needs. For ordinary domestic consumers the voltage is again reduced to 220 V.

Exercises

14.1 What are the dimensions of magnetic flux ?

14.2 Mark the current direction in the secondary windings of Fig 14.36 as the switch is closed.

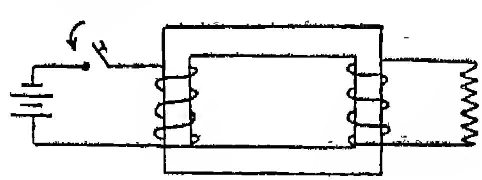


Fig. 14.36

Amplitude of voltage is,

$$2\pi f L I_0 = 2\times3.14\times50\times1.0\times0.5 = 157 \text{ volts}$$

$$V = 157 \cos 100\pi t$$

Therefore,
The voltmeter will read the effective value

$$V_{eff} = \frac{V_0}{\sqrt{2}} = \frac{157}{\sqrt{2}} = 112 \text{ volts.}$$

EXAMPLE 14.7

What is the inductive reactance of a coil if the current through it is 80 mA and voltage across it is 40 V ?

Solution

These are the effective values hence,

$$X_L = \frac{V_{eff}}{I_{eff}} = \frac{40}{80\times10^{-3}} = 500 \text{ ohms.}$$

EXAMPLE 14.8

At what frequency will a 0.5 henry inductor have a reactance of 2000 ohms ?

Solution

$$X_L = \omega L = 2\pi f L$$
$$2000 = 2\pi f\times0.5$$
$$f = \frac{2000}{\pi} = 637.$$

14.11. A. C. Circuit Containing Capacitance Only

A capacitor consists of two plates of conducting material separated by an insulator. Its resistance is, therefore, practically, infinite. We might expect that when a capacitor is put into a circuit, d. c. or a. c., no current would flow. This is not so. Let us first consider a capacitor in a d.c. circuit.

Capacitor in a d c. circuit : Consider the circuit shown in Fig. 14.24. As soon as we

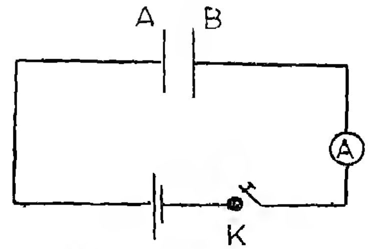


Fig. 14.24 Capacitor in a d.c. circuit

press the key, electrons begin to flow from the negative terminal of the battery to the plate B and from plate A to the positive terminal of the battery. The plate A thus begins to acquire positive charge and the plate B, negative charge. This charging of the capacitor continues till the potential difference between the plates becomes equal to that across the battery terminals. Then, it ceases. This flow of charge is equivalent to a current. Thus a current does flow in the circuit, during the charging process, though not through the capacitor but in the remainder of the circuit. The ammeter would, therefore, show a momentary deflection. The direction of current is from the plate B to the plate A via the battery. Its magnitude at any instant is given by the rate of growth of charge on the capacitor.

$$I = \frac{dQ}{dt} \quad \text{... (14.29)}$$

The charging process can be extended in time if we include a resistance in the circuit, Fig. 14.25. It is found that the final charge on the capacitor, given by

$$Q_0 = V_0 C \quad \text{.. (14.30)}$$

where, V_0 = battery voltage, C = capicitance, is reached asymptotically as shown in Fig. 14.26. (Compare with the growth of current in a d.c inductive circuit).

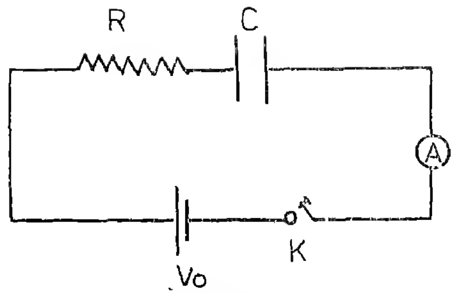


Fig. 14 25 Charging of a capacitor through a resistance

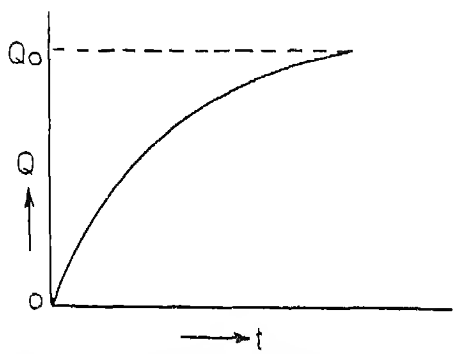


Fig. 14.26 Growth of charge on capacitor with time

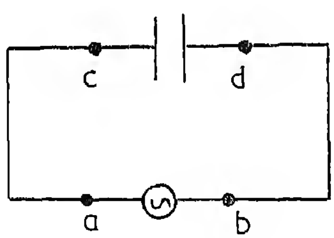


Fig. 14.27 A C source with a capacitor

Capacitor in an a c. circuit: Now consider the capacitor in an a c. circuit, Fig 14.27. As the voltage from the source is continually changing, the charge on the capacitor is also continually changing During a complete cycle, the capacitor is first charged in one direction, then discharged, again charged in the reverse direction and discharged. As this charging and discharging of the capacitor is taking place continuously, a continuous current exists in the circuit. Let us find out the nature of this current.

It is evident that the p d. across the capacitor between points c and d, at every instant, has to be exactly the same as that across the source terminals a and b. Therefore the capacitor must charge and discharge in such a manner that the p.d. across it, V, is sinusoidal and equal to the applied e m f at every instant. That is,

$$V = E = E_0 \sin \omega t$$

The charge on the capacitor, at any instant, is given by

$$Q = CV$$

Therefore, current at any instant is given by

$$\begin{aligned} I &= \frac{dQ}{dt} = C\frac{dV}{dt} = C\frac{d}{dt}(E_0 \sin \omega t) \\ &= E_0 C\omega \cos \omega t \\ &= I_0 \cos \omega t \\ &= I_0 \sin (\omega t + \pi/2) \qquad \text{... (14.31)} \end{aligned}$$

where,

$$I_0 = \frac{E_0}{1/C\omega} \qquad \text{... (14.32)}$$

Thus, in this case, the current is sinusoidal but 90° ahead of e.m f., in phase. The wave form of E and I are shown in Fig. 14.28. The quantity

$$X_C = \frac{1}{C\omega} = \frac{1}{2\pi f C} \qquad \text{... (14.33)}$$

is known as the capacitive reactance of the circuit. It plays the same role in this case, as the inductive reactance X_L in the inductive case. Its unit is ohm.

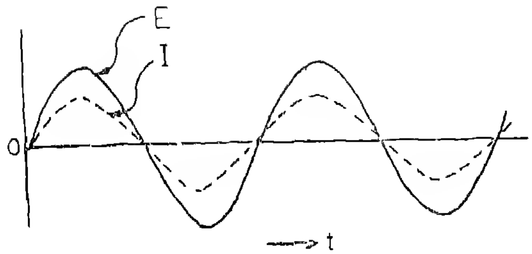


Fig. 14.28 Current I leads the voltage E by $\pi/2$ in a capacitive circuit

A voltmeter connected across the capacitor would read $E_0/\sqrt{2}$ for the effective voltage and an ammeter in the circuit would read $I_0/\sqrt{2}$ for the effective current We, then, have

$$I_{eff}=\frac{I_0}{\sqrt{2}}=\frac{E_0}{\sqrt{2}\,(1/C\omega)}=\frac{E_{eff}}{1/C\omega}=\frac{E_{eff}}{X_C} \quad .\ (14\ 34)$$

Example 14.9

What is the capacitive reactance of a 5-μf capacitor when it is part of a circuit whose frequency is (*i*) 50c/s (*ii*) 10^6c/s ?

Solution

(*i*) $X_C=\dfrac{1}{2\pi\times 50\times 5\times 10^{-6}}$ ohms

$= 637$ ohms

(*ii*) $X_C=\dfrac{1}{2\pi\times 10^{6}\times 5\times 10^{-6}}$

$=3.18\times 10^{-2}$ ohms.

14 12 LCR Circuit

A resistor, an inductor, or a capacitor, each of these circuit elements impedes an alternating current. The impeding effect is measured by the resistance (R), the inductive reactance (X_L) and the capacitive reactance (X_C), respectively, which are defined by

$$\frac{V}{I}=R \text{ (for resistor)}$$

$$\frac{V}{I}=X_L \text{ (for inductor)}$$

$$\frac{V}{I}=X_C \text{ (for capacitor)}$$

where V is the effective voltage across the circuit element and I the effective current through it.

Where a combination of these elements forms part of a circuit the total current impeding effect is defined, in an analogous manner by,

$$Z=\frac{V}{I} \quad ..\ (14.35)$$

where Z is known as the impedance of the combination Its unit, obviously, is ohm.

In Fig. 14.29 we have a series combination of a resistor of resistance R, an inductor of reactance X_L and a capacitor of reactance X_C. It turns out that the total impedance Z of the combination is given by

$$Z=\sqrt{R^2+(X_L-X_C)^2} \quad \ldots\ (14.36)$$

and not by $Z=R+X_L+X_C$. The reason for this peculiar relationship is the fact that the reactances (X_L, X_C) arise because of a somewhat different physical mechanism than ordinary resistance (R). We cannot, therefore, combine them by simple addition. Equation (14.36) is a direct consequence of the vectorial addition of effective voltages in an a.c. circuit as discussed below.

In the circuit of Fig. 14.29, at any instant, the applied voltage is equal to the sum of the voltage drops across individual elements.

$$V=V_R+V_L+V_C \text{ (instantaneous values)}$$

However, if we measure the effective voltages

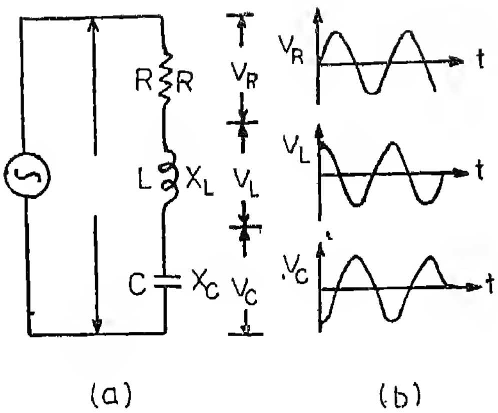


Fig. 14.29 An LCR series circuit

(with a voltmeter) we would find that

$$V \neq V_R + V_L + V_C \text{ (effective values)}$$

The reason is, although same current passes through each component, the voltage across each of them bears a different phase relationship with the current and so the voltages are out of phase with one another. Mathematical analysis shows that effective voltages (and currents) add up vectorially. It is customary to treat them as vectors for the purpose of circuit analysis. Phase differences are represented by angles between the vectors.

Thus, in Fig. 14.30, the current I is represented by the vector OI; V_R, the voltage across R, being in phase with the current, is represented by vector OR; V_L, the voltage across L, being ahead of current by $\pi/2$, by vector OL and V_C, the voltage across C, since it lags behind the current by $\pi/2$, by vector OC. The resultant vector OP represents the effective voltage across the combination Its magnitude is given by,

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} \qquad (14.37)$$

Since $V_R = RI$, $V_L = X_L I$, $V_C = X_C I$ we have

$$V = \sqrt{R^2 I^2 + (X_L - X_C)^2 . I^2}$$

$$= \sqrt{R^2 + (X_L - X_C)^2} . I$$

or $$\frac{V}{I} = Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad (14.38)$$

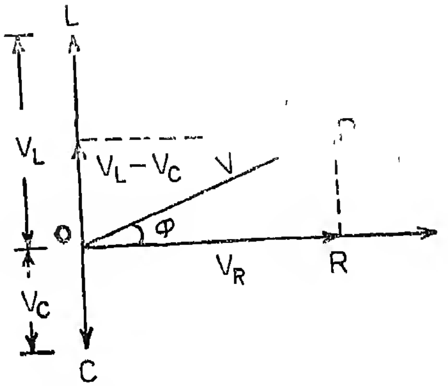


Fig. 14.30 Vectorial representation of voltages in an a.c circuit

The relationship between impedance (Z), resistance (R) and reactances (X_L, X_C) is illustrated in Fig. 14.31 by constructing a vector diagram similar to Fig. 14.30.

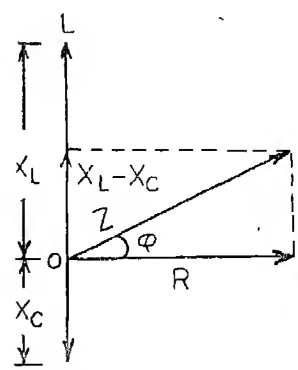


Fig. 14.31 Impedance diagram

density. The mean densities of dwarfs stars range from 10 Kg/m^3 for the hottest O stars to 50000 Kg/m^3 for the coolest M stars. The lowest densities are found in supergiants e.g Antares (α Scorpii) is only three thousandth as dense as air. The densest stars are white dwarfs like the companion of Sirius which is 5000 time denser than the densest element platinum. It is often said of these stars that one matchbox full of their material will weigh one ton on the surface of the earth.

As in the case of the sun, temperature and density increase towards the centres of stars. The central densities of dwarf stars range from a few thousand Kg/m^3 for O type stars to 10^4 Kg/m^3 for F,G,K and M type stars. The central temperature is maximum (about 30 million degrces) for early type O and B stars and minimum (about 10 million degrees) for M type stars.

15.8 Stellar Evolution

Like every thing else in the universe stars are born live for a certain length of time and die at the end of their career. The life history of a star begins when a large cloud of interstellar dust and gas begins to contract under the influence of its own gravitational force. It is calculated that the cloud must have a mass of at least one thousand solar masses for this to occur. As the cloud contracts it heats up by compression A part of the energy is radiated away, which helps the cloud to contract further. At some stage the cloud breaks up into a large number of fragments of stellar size and each fragment continues to contract. When fragments become hot enough, they begin to radiate light from their surface. Thus each fragment becomes a self-luminous star and a cluster of stars is born.

Each star of the cluster continues to contract until the central temperature attains values of ten million degree or more. At these temperatures the thermonuclear reactions discussed in section 15 5 start. The carbon-nitrogen cycle and the proton-proton chain reaction convert hydrogen into helium and the energy liberated in these processes keeps the star shining for millions of years. The dwarf stars, which form more than 90% of the whole stellar population, are found to be in this stage of evolution. It lasts until all hydrogen fuel in the central ten per cent core of the star is exhausted The more massive stars of O and B type spend their fuel most rapidly like a spendthrift within 10 to 100 million years The least massive M type stars, which are more frugal, can remain shining for more than thousand billion years. The sun has a total life of 10 billion years out of which about half is already over at the present time.

When the hydrogen in the core of the star is exhausted the core begins to contract while the outer regions expand. In this way the radius of the star increases while the surface temperature drops and the star becomes giant or supergiant. For the sun this stage will occur about 5 billion years from now.

The time spent by a sta in the giant and supergiant stages is small compared to the duration of the dwarf phase. That is why we find most stars in the dwarf stage and only a few in the evolved stages or giants and supergiants. At the end of the latter stage the production of energy in the star is so large and rapid that the star explodes in the form of a nova or supernova throwing out a large portion of its envelops into interstellar space. The core that remains behind may end up as one of the following three types of stellar remnants or corpses :

(i) If the original mass of the star was less than about 2 solar masses we get a dense white dwarf of less than 1.2 solar mass. As there is no nuclear fuel left in the white

dwarf it just cools off slowly changing its colour from white to yellow, to red and finally becomes a dark body.

(ii) If the original mass of the star was between 2 and 5 solar masses the back kick of the supernova explosion will compress the core of the star to nuclear densities giving rise to a neutron star. The mass of a neutron star is less than 2 solar masses and its radius is about 10 kilometres. Neutron stars have very large magnetic fields of the order of 10^{12} gauss. If the magnetic axis is inclined to the axis of rotation, the star emits pulses at regular intervals, the periods of which range from 30 milliseconds to 3 seconds. These are pulsars the first of which was discovered by the radio astronomers in 1967.

(iii) If the original mass of the star was more than 5 solar masses, the back kick of the supernova explosion is so violent that the core continues to contract indefinitely giving rise to a black hole. As the contraction proceeds, the radius decreases continuously and acceleration due to gravity, g, at the surface goes on increasing. Finally a stage comes when the g value is so large that even the photons cannot escape from the surface of the body. On the other hand any particle or photon approaching it will be immediately swallowed. That is why such a body is called a black hole. The recently discovered X-ray source Cygnus-XI is found to be a binary system in which one component is believed to be a black hole.

15.9 The Milky Way

(*a*) *Size and shap* : We have noted earlier that our solar systems and most of the nakedeye stars are members of a very large system which appear to us as Milky Way (*Akashaganga*) in the sky. It is a flat lens shaped disc which is thicker near the centre and thins out towards the edges. We are situated very nearly in the central plane of the milky way. That is why it appears to us as a great circle in the sky. But the milky way is not uniformly bright everywhere. It is brighter and broader in one hemisphere, and less conspicuous in the other. This is due to the fact that the sun is not at the centre of the galaxy, the centre lies in the direction

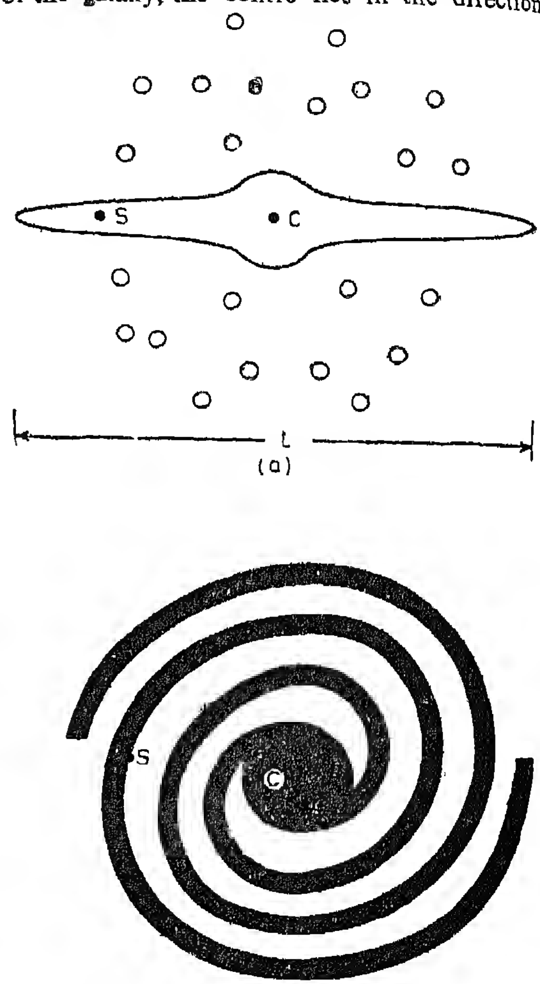


Fig. 15.8 (a) Schematic view of Milky Way i.e. Edge on view C=galactic centre S=position of Sun
(b) Schematic view of Milky Way i.e. Face on view C=galactic centre E=position of Sun

of the Sagittarius (*Dhanur Rashi*) constellation. The diameter of the galactic disc is 100000 light years and the sun is situated at a distance of 30000 light years from the centre. The thickness of the disc is 5000 light years at the centre, about 1000 light years near the sun, and diminishes as we reach the edges (Fig 15.8).

(*b*) *Interstellar matter* : On a clear night we see several dark regions in the band of the milky way. These regions look dark not because there are fewer stars in those directions, but on account of the intervening dust clouds which hide from our view the stars lying behind them. Dust and gas occupy the spaces between the stars and sometimes form dense clouds containing 10_8 to 10^9 atoms per m^3. But these densities are minute compared to the density of our earth's atmosphere. Like fog on the surface of the earth the dust in the interstellar space diminishes the light of the stars behind them And just as the sun looks red at the time of rising and setting because its light has to pass through a great thickness of our atmosphere, similarly the light of the star becomes reddened as it passes through the dust clouds which lie in its path. Sometimes a dust cloud is illuminated by a hot star and it begins to shine by the reflected light giving rise to a bright nebula like Orion nebula. The gas which forms 90 per cent of the mass of the interstellar matter can be recognised by its characteristic spectrum lines which appear either in emission or in absorption. Hydrogen is the most abundant of all the elements and it gives rise to an emission line in the radio region of the spectrum; it is called the 21 cm line which is very useful for locating the hydrogen clouds in our galaxy.

(*c*) *Star clusters* : The galaxy also contains smaller assemblies of stars known as clusters. Two kinds of star clusters are recognised. The open clusters, of which the Pleiades (*Krittika*), Hydes (*Rohinishakata*), Praescape (*Pushya*) are good examples, contain 100 to 1000 stars which appear separate on photographs Quite distinct from galactic clusters are globular clusters which derive their name from their spherical appearance. A globular cluster contains about 100000 stars which are packed rather closely together. About 100 globular clusters are known, all of them are more than 20000 light years away from the sun.

(*d*) *Structure* : It is now known that the disc of our milky way has a spiral structure like what we see in the Andromeda galaxy (Fig. 1 1). Ordinary optical telescopes are inadequate for the study of its detailed structure Our view is cut off by the interestellar dust clouds so that only a small portion in the solar neighbourhood can be surveyed. But radiowaves can penetrate through the dust clouds; therefore a detailed study of the galactic structure has been made in recent times by observations of the 21 cm line of hydrogen with radio telescope.

The central part of our galaxy looks like an amorphous spheriod and it resembles a huge globular cluster in many ways. Outside this central nucleus are situated the spiral arms which are about 1200 light years wide and are separated by gaps of about 500 light years. The sun is situated near the inner edge of one such arm which passes through the Orion nebula. The spiral arms contain the youngest objects in the milky way. They include the gas and dust, the open clusters and the hot blue stars. The inter-arm regions contain somewhat older stars But the oldest stars are found in the galactic halo and in globular clusters. Fig. 15.8 shows schematically two views of our galaxy, one edge on and the other face on.

(*e*) *Rotation and mass* : The milky way is not stationary, it is rotating around an axis

passing through its centre. This is the real reason for its flat disc like shape The globular clusters do not take full part in rotation, hence they form a spherical halo around the galaxy. The galaxy does not rotate like a solid wheel. Each star revolves around the central nucleus in an elliptical or circular orbit just like the planets round the sun. The stars near the centre move faster than the stars farther out. Our sun moves round the centre of the galaxy with a speed of 250 km/sec. The sun takes about 240 million years to complete one revolution around the galactic nucleus, which can be called the galactic year. Here also, equating the centrifugal force caused by the sun's circular velocity with the gravitational force of attraction by the central nucleus we can write: M (Galaxy)$=a\,v^2/G$. Putting $a=30{,}000$ light years$=3\times10^{20}$m, $v=2.5\ 10^{5}$m/sec. and $G=6\,67\ 10^{-11}$ MKS units, we find that the mass of the galaxy is 3×10^{11}kg$=150\times10^{9}$ solar masses. Hence, if we assume that the galaxy is made of average stars like the sun we can conclude, tha there are 150 billion stars in our galaxy.

15 10 Galaxies and the Universe

(*a*) *Normal galaxies*: Stellar systems like our Milky Way form the major building blocks of the universe; they are called galaxies. Hundreds of millions of galaxies have been photographed by the modern powerful telescopes. The farthest of them are about a billion light years away.

All galaxies do not look alike, and on this basis they are divided into three classes. About 3 per cent of all galaxies are irregular in shape, they are called irregular galaxies. A large number of galaxies show spiral arms, they are known as spiral galaxies Our Milky Way and the Andromeda galaxy are good examples. Most common are elliptical galaxies which show elliptical discs on photographs.

(*b*) *Radio galaxies and quasars*: Normal galaxies emit a moderate amount of radio radiation compared to their light output. But certain radio sources in the sky are found to emit millions of times more energy in the radio region compared to normal galaxies. They can be divided into two types; radio galaxy and quasars Radio galaxies are identified with some peculiar optical galaxies showing explosions in them. But the radio radiation does not come from the galaxy itself. Invariably we find two radio sources occurring symmetrically on either side of the peculiar galaxy, like two ears on the two sides of the face of a man (Figure 15.9). It is

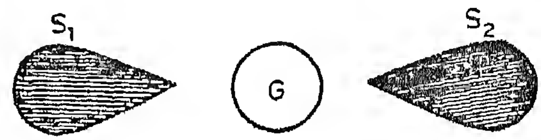


Fig. 15.9 A=Radio galaxy G=Central optical galaxy S_1,S_2=Two parts of radio source

believed that a great explosion occurred in the central galaxy and two clouds of charged particles were ejected on two dimetrically opposite sides.

The other kinds of powerful radio sources are identified with star-like objects which are called quasi-stellar radio sources or shortly as QUASARS. The quasars are found to have radial velocities of recession equal to 90 per cent of the velocity of light. Consequently they are placed at large distance exceeding the distances of the most distant galaxies. The cause of the tremendous energy output of these star-like objects is still a mystery. However, their large distances make them important for solving the problems of cosmology discussed in the next section.

(*c*) *The expanding universe*: One

important feature of the galaxies is that they all appear to move away from us. The speed of recession 'v' increases proportionately with their distance 'r'. We can write v=Hr which is known as Hubble's law. 'H' is known as Hubble's constant, it is found to have a value of 16000m/sec for every million light years. It is with the application of Hubble's law that we find the distances of objects like radio galaxies and quasars. Hubble's law indicates that the universe of galaxies is expanding. Based on these facts of observation several cosmological theories about the origin, and evolution of the universe are put forward.

According to the 'big bang' theory, all matter in the universe was concentrated in a very dense and hot primeval fireball at the beginning. An explosion occurred at about 20 billion years ago and since then the matter in the universe is moving away in the form of galaxies Using the value of the Hubble's constant we find that the velocity of expansion equals the velocity of light at a distance of 20 billion light years. Since the light from the galaxies at that distance can never reach us it is called the boundary of the observable universe. On account of the continuous expansion more and more galaxies will go beyond this boundary and they will be lost. Consequently the number of galaxies per unit volume will go on decreasing and ultimately we will have an empty universe.

If the total mass of the universe is more than a certain value the expansion may be stopped by its gravitational pull and the universe may contract again. Thus we may have alternate expansion and contraction giving rise to a pulsating universe. There is also a third possibility suggested by T Gold and F. Hoyle in U.K. They postulate that new galaxies are continuously being created out of empty space to fill up the gap caused by the galaxies which leave the observable part of the universe. This is known as the 'steady state theory.'

The presently available observations of galaxies and quasars are not sufficient to decide which of three theories of cosmology is the correct one. However, one piece of independent observation is against the steady state theory. It is observed that radio radiation of a few millimetre wavelength is coming from all parts of the sky. This background radiation has a temperature of 3K and it is believed to be the remnant of the original very hot radiation of 10^{10} K which existed at the time of the big bang But the question whether the universe will continue to expand for ever or whether it is a pulsating universe is still not settled. We need to know the average density of the universe much better before a final answer can be obtained.

Table 1

PHYSICAL PROPERTIES OF THE OBJECTS IN THE SOLAR SYSTEM

Object	Period of revolution in years	'a' in A.U.	Radius R/R(Earth)	Rotation period	Mass M/M(Earth)	Mean density (kg/m × 10⁻³)	g/g(Earth)	Temperature (Degree C)	Albedo	Surface pressure in atmospheres	Atmospheric chemical composition	No. of satellites etc
Moon	—	—	0.27	27.32 d	0.0123	3.34	0.170	+110 (day) —150 (night)	0.07	0	Vacuum	—
Mercury	0.241	0.387	0.38	58.6 d	0.056	5.4	0.367	+340 (day) —120 (night)	0.06	0	Vacuum	0
Venus	0.615	0.723	0.96	243.0 d (retrograde)	0.815	5.1	0.886	+480 (surface) —40 (clouds)	0.85	100	CO_2 (95%)	0
Earth	1.000	1.000	1.00	23h56m 1	1.000	5.52	1.000	+45 (Equator) —50 (Poles)	0.40	1	N_2 (80%) O_2 (20%)	1
Mars	1.881	1.524	0.53	24h27m4	0.107	3.97	0.383	+30 (Eq.) —130 (Poles)	0.15	0.006	CO_2 (97%) N_2 (3%)	2
Ceres (Largest Asteroid)	4.603	2.767	0.055	9h05m	0.0001	3.34	0.18	—	0.07	0	Nil	++
Jupiter	11.864	5.203	11.23	9h50m 5	317.9	1.33	2.522	—140 (Clouds)	0.45	—	H_2, He, CH_4, NH_3	14
Saturn	29.46	9.540	9.41	10h14m	95.2	0.70	1.074	—175 (Clouds)	0.61	,,	H_2, He, CH_4, NH_3	10+ring
Uranus	84.01	19.18	3.98	10h49m (retrograde)	14.6	1.33	0.922	—220 (Clouds)	0.35	,,	H_2, He, CH_4	5+ring
Neptune	164.1	30.07	3.88	15h	17.2	1.66	1.435	—230 (Clouds)	0.35	,,	H_2, He, CH_4	2
Pluto	247	39.44	0.5	6.39 d	0.11	4.9	0.440	—240 ?	0.14	?	?	Nil
Halley's comet	76.2	17.8	+	—	10^{-13} 10^{18} g		—	—	—	—	—	—

Notes : 1 year=365.257 days, 1 A.U.=1.496 × 10^8 Km; R (Earth)=6378 Km, M (Earth)=5.977 × 10^{27} g, g (Earth)=9.82 m/sec^2,
+Comet Nucleus diameter 10 Km, head 10000 Km, ++ Total number of asteroids > 1600
d=day h=hour m=minute

Table 2

PROPERTIES OF STARS OF VARIOUS SPECIAL TYPES

Spectral type	Colour	Example	Description of Spectrum	Temperature	$\frac{M}{M (Sun)}$	$\frac{R}{R (Sun)}$	Absolute Magnitude Mv	$\frac{L}{L (Sun)}$
O	Very Blue	θ Orionis	Lines of ionised Helium	35000	40	20	—6 0	10^5
B	Blue	Spica (Chitra)	Lines of neutral Helium	20000	15	7	—3 1	10^4
A	White	Sirius A (Vyadha)	Balmer lines of Hydrogen	9500	2 3	1.8	+1 4	25
F	Green	Procyon	Lines of Hydrogen and ionised metals	7000	1.4	1 2	+2 7	3
G	Yellow	Sun (Surya)	Lines of ionised and natural metals	5800	1 0	1 0	+4.8	1
K	Orange	Σ Eridani	Lines of neutral metals and molecular bands	4500	0.7	0 8	+6.1	0.4
M	Red	Kruger 60	Molecular bands of Tio	3500	0 3	0.4	+11 8	1/40
M	Red	Betelgeuse (Ardra)	Tio bands	3000	20	220	—6	5×10^4
A	White	Sirius B	Broad lines of hydrogen	9500	1 0	1/50	+11.5	1/200

Exercises

15.1 (a) Give a list of various constituents of the universe.

(b) How does the science of astronomy differ from other sciences like Physics ?

15.2 (a) Describe and compare the various kinds of telescopes used for astronomical observations.

(b) The lens of our eye has a diameter of 8 mm. How much fainter objects can be seen through a telescope of 120 cm aperture as compared to the faintest naked eye stars.

15.3 (a) The distances of the satellites of Mars are 25″ for Phobos and 62″ for Deimos at mean opposition when Mars is 0.524 A.U. for earth. Calculate distances of the two satellites for Mars in astronomical units and in metres.

(b) If the period of revolution is 0.319 day for Phobos and 1.262 day for Deimos use equation (15.3) to find the mass of Mars in units of the earth's mass which is 5.977×10^{24} kg.

15.4 (a) Explain why some bodies in the solar system have atmospheres and others do not.

(b) Discuss the possibility of life on the various planets in the solar system.

15.5 Period of revolution of the moon is 27.32 days and its mean distance from earth is 384,400 km. Use equation (15.2) to calculate the sum of the masses of the earth and moon. Further, using the known fact that the centre of mass of the earth-moon system lies at 4.75×10^{6}m from earth's centre, calculate the mass of each.

15.6 (a) Calculate the value of the solar constant at the distance of Jupiter which is 5.2 A.U away from the sun. Hence calculate the temperature of a black body on Jupiter by Stefan's law.

(b) Explain how the sun produces its energy.

15.7 (a) From the data given in columns 4 and 6 of Table 1, calculate and verify the entries about the mean density and acceleration due to gravity given in columns 7 and 8.

(b) A man weighs 68 kg on earth. Find his weight on the various objects in the solar system.

15.8 From the data in Table 2, calculate the mean densities and g-values on the surfaces of various types of stars.

15.9 (a) The two components of Procyon have a separation of 4″.55 and period of 40 6 years. If the distance of the binary is 11.3 light years, calculate the sum of the masses of the two stars in solar units. If the ratio of their masses is 3:1 find the mass of eachs tar.

(b) If both components of Procyon have the same spectra while one component is 15000 times brighter than the other, calculate the ratio of their radii.

15.10 Narrate the complete life history of a star

15 .11 Describe the structure and contentsof the Milky Way.

15.12 (a) Give the properties of normal galaxies and explain how the radio galaxies and quasars differ from them.

(b) Briefly discuss the three main cosmological theories of the universe.